Classroom Activities for Digital Interactive Simulations to Support Realistic Mathematics Education

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Paper presented at the 6th International Realistic Mathematics Education Conference

Georgetown, Cayman Islands

September 20, 2018

Abstract

Digital interactive simulations have been used by the Realistic Mathematics Education community and elsewhere to support student sense making and conceptual understanding in mathematics. In this study, we use an RME lens to examine a new set of math interactive simulations - PhET simulations – the accompanying sim-based lesson activities, and the reflections of students and teachers who have used these resources in their middle school classrooms. Analysis of the simulation and lesson design demonstrates significant alignment to RME's activity, reality, and level principles and ample opportunity for teachers to apply these principles to their teaching. Analysis of post-study teacher interviews and student post-lesson reflections find that these principles were not uniformly taken up by teachers and students. Both students and teachers appreciated the opportunities for activity in the lessons. Students enjoyed the realistic and visual components of the simulation while only some teachers discussed the realistic nature of the simulations, and those that did, discussed it as a differentiation tool rather than a uniform benefit. Teachers also expressed an appreciation for how the sim-based lessons supported progressive formalization; however, some students thought these connections were lacking. The paper concludes with implications and recommendations for researchers and practitioners.

Introduction

From the earliest days of the personal computer, there have been proposals for using technology in ways that can support mathematics education. Nearly 30 years ago, the Mathematical Sciences Educational Board (1989) made the broad claim that “the changes in mathematics brought about by computers and calculators are so profound as to require readjustment in the balance and approach to virtually every topic in school mathematics” (p. 2). Given the history of mathematics as a field of study in education psychology, mathematics education has been a discipline of interest when investigating ways computers and related technology might support student learning. With the advent of more sophisticated and interactive graphical user interfaces, dynamic realistic representations have created new opportunities for teachers and students to explore and understand mathematics.

Visual, interactive, dynamic mathematics software such as The Geometer’s Sketchpad (Jackiw, 2014), SimCalc (Roschelle & Kaput, 1996), GeoGebra (Hohenwater, 2001), and Tinkerplots (Konold, 2007) have increased opportunities for students to learn concepts in mathematics classrooms. These instructional resources for mathematics education not only have experienced an extended period of iterative development and design, but they have also demonstrated some degree of scalability in mathematics education around the world. Other resources, such as the Freudenthal Institute’s WisWeb (Boon & Drijvers, 2005; van Reeuwiejk, 2002) and the National Library of Virtual Manipulatives (Dorward & Heal, 1999), have organized collections of standalone interactive applets that can be used to illustrate specific concepts and practice skills with individualized feedback. In some cases, learning management systems have been used to organize applets and tasks into instructional sequences that allow for sustained exploration of a topic and the ability to save and return to prior work (e.g., the Digital Mathematics Environment, now hosted by Numworx). It is also worth noting that the role of technology in mathematics education, and related development of supporting software, has been explored extensively by researchers at the Freudenthal Institute in primary grades, as students interact with mathematics applets characterized as digital games (e.g., Bakker, Van den Heuvel-Panhuizen, & Robitzsch, 2016; Jonker, Wijers, & van Galen, 2009). Similar lines of inquiry on the role of technology in secondary mathematics education have also been pursued (e.g., Bakker, A. & Derry, J. 2011; Doorman & van der Kooij, 1992; Drijvers, Doorman, Kirschner, Hoogveld, & Boon, 2014)

In this paper, we examine how a collection of digital simulations - specifically PhET simulations - and associated sim-based lessons align with and support principles of Realistic Mathematics Education. We first examine the design of the simulations and lessons through an RME lens, highlighting design features across three simulations and their connection to RME principles. We also analyze student and teacher reflections about their experiences in teaching and learning with simulations - data that was collected as part of a larger study of the use of PhET simulations in middle school mathematics classrooms. Our analysis and discussion focuses on the activity, reality, and level principles. We conclude with implications and recommendations for researchers and practitioners.

PhET Interactive Simulations

Since 2002, the PhET Interactive Simulations project at the University of Colorado Boulder has focused on the design and study of interactive simulations for teaching and learning science (Wieman, Adams, & Perkins, 2008). More recently, the PhET team partnered with math education researchers to expand its efforts, creating a growing suite of interactive simulations for mathematics instruction. The collection of 38 simulations - including 27 simulations creating highly interactive math contexts for learning and 11 simulations creating mathematics learning opportunities in science contexts - cover a range of math topics and grade levels. Licensed as open educational resources, PhET's mathematics simulations are used millions of times per year, worldwide.

The design principles for PhET's mathematics simulations are derived from goals for student learning that are consistent with NCTM's (2000) Principles and Standards for School Mathematics charge that "students must learn mathematics with understanding, actively building new knowledge from experience and previous knowledge" (p. 2). By increasing opportunities for student sense making and knowledge construction, students will have greater access to and increased ownership of mathematics. Hensberry, Paul, Moore, Podolefsky & Perkins (2013) also argued that the most recent articulation of mathematics standards in the United States, the Common Core Standards for School Mathematics ("Common Core State Standards Initiative. Common Core State Standards for Mathematics," 2010), is well aligned with instructional goals for PhET mathematics simulations. Written from a student perspective these instructional goals are: To see mathematics as accessible, understandable, and enjoyable; to make connections to everyday life; to achieve conceptual understanding; to engage in mathematical exploration with multiple, positive learning outcomes; and to take and sense ownership of their learning experience (Hensberry, Moore, & Perkins, 2015, p. 150). It is worth noting that these goals reflect both affective, cognitive, and identity goals for students.

Over the years, the PhET team has developed a research-based set of simulation design principles, with the goal of creating tools that flexibly and powerfully support these student goals. Described by Hensberry et al (2013), PhET's math simulation design principles are: 1. Use implicit scaffolding; 2. Make simulations highly interactive; 3. Provide dynamic feedback; 4. Use multiple, linked representations; 5. Allow pedagogically powerful actions, even if difficult or impossible in real life; 6. Provide an interactive interface; 7. Emphasize real world connections; 8. Create a game-like environment that is engaging, fun and open (pp. 151-152). Implicit scaffolding is a particularly central principle, and focuses on how, through its design, a simulation can implicitly invited and guide students' natural interactions with and sense making around various mathematical objects, tools, and options in each sim. As described by Podolefsky, Moore, and Perkins (2013):

> When students begin attempting to answer their questions through interaction with the sim, we want the sim design to enable students to engage in productive inquiry – where their attempts at sense making with the sim generate constructive questions and ideas, followed by further exploration for answers. To do this, we design the overall layout, representations, feedback, range of interactions and illuminating cases to support productive sense making (p. 9).

Team discussions of sim features from a student (and teacher) perspective are common and necessary to build a robust set of features that can behave as instructional scaffolds for students that motivate individual play and exploration without much additional instructional overhead (Webb, 2018).

Importantly, the student and teacher experience in the classroom is shaped not only by the simulation, but also by the design of the paired lesson and approach to teacher facilitation. PhET's sim-based lesson design principles emerged from work with science (Perkins, Moore, Podolefsky, Lancaster, & Denison, 2012) and math (Hensberry et al., 2013). The importance of student exploration, which can also be supported by peer interaction, is emphasized as an important introductory component of PhET lessons, described as "open play." As students make sense of relationships between mathematical objects in a sim, they are encouraged to propose observations and conjectures. Ideally, teachers would then use students' ideas to influence follow-up questions and other aspects of the lesson.

Realistic Math Education Principles

Realistic Mathematics Education is a response to the behaviorist assumptions about the teaching and learning of mathematics, and direct teaching of formal mathematics, that was prevalent during the late 1960s. As recounted by van den Heuvel-Panhuizen and Drijvers (2014), "In the 1960s, mathematics education in the Netherlands was dominated by a mechanistic teaching approach ... taught directly at a formal level … and procedures [were learned] step by step with the teacher demonstrating how to solve problems" (pp. 521-522) RME was a reaction to this approach and the potential influence of "New Math" that was prominent in the United States, France and the United Kingdom during that time. In these behaviorist mathematics classrooms students were trained to repeat learned algorithms in response to various abstract math problems. Proponents of Realistic Mathematics Education have argued that this operational approach lacks connection to the students' lived experiences, and that lack of student understanding of how mathematical principles work is at the core of why these approaches to mathematics education have led to "inflexible and reproduction-based knowledge in mathematics students" (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 522).

Activity Principle

In Realistic Mathematics Education, students are positioned as active learners in the process of doing mathematics, an approach that aligns with other reform based approaches to teaching mathematics. A study of student use of fractions in normative context found that inquiry based classroom practices that build on student thinking (one form of reform based teaching) were more effective at developing student use of fraction notation (Saxe et al., 2005). In Realistic Mathematics Education, students are empowered as learners in an iterative process through which they form their own constructions and productions (Streefland, 1991). Students need to form their own understandings and construct meaning behind what they are doing, not just be told how a process works and expected to repeat it in new contexts when needed.

Reality Principle

RME focuses on creating realistic contexts for problems where students can explore mathematical principles instead of just being shown worked examples and being expected to practice similar examples individually. The "realistic" part of RME connects to both "real world" scenarios but also the idea that problem contexts are imaginable, and thus real in your mind. Regardless of level of how “real” the problems are, they are designed to motivate mathematical strategies (Webb, van der Kooij, & Geist, 2011). This does not exclude imaginary or abstract contexts or subjective ‘pieces of reality’ as Hans Fruedenthal calls it (2002 p. 67). The “real” world in an RME context can include lived experience, imagined fantasy or the formal world of mathematics. Connections between their lived or imagined experiences and the abstract concepts of mathematics help students formalize their understandings of mathematics.

Level Principle

Realistic Mathematics Education also includes the idea of progressive formalization, the process of “going from the concrete to the abstract” (Dewey, 1910) and gives students the ability to form new mathematical knowledge, connecting the concrete and abstract. Moving students’ understandings of mathematics from a purely informal and experience (or imagination) based context to a more abstract and formal context is one of the primary goals of Realistic Mathematics Education.

Study Context

This paper investigates how, from a Realistic Mathematics Education perspective, digital interactive simulations and sim-based lessons help support teachers and students in the learning and teaching of middle school mathematics. Results from a pilot study using PhET simulations in middle school classrooms are analyzed to help answer the following questions:

1. How are RME principles supported in the design of PhET simulations and sim-based lessons?
2. How are RME principles evident (or not) in student and teacher reflections about learning mathematics with PhET simulations?

Data for this paper comes from a pilot study that investigates how PhET interactive simulations (and the associated lesson activity sheets) can support the development of mathematical reasoning and promote student engagement in middle school mathematics classrooms. The design team included several RME scholars who were influential in the design of both the simulations and the activity sheets used in this study. Some activity sheets were written by the research team and some were modified from lessons written by teacher partners to align to the curriculum prescribed by the district enrolled in the study.

First, select simulations and activity sheets are described to show how the RME principles are evident in their design. Through this discussion we present the intent of the designers (several are co-authors of this paper). Next, we analyze data from students and teachers using PhET simulations in their classrooms. Analysis of teacher and student perspectives investigates how the RME principles are apparent in their perspectives on using the PhET simulations and activity sheets.

Methods

During the 2017-18 school year, six middle school teachers across 4 schools participated in the study. These teachers had no prior experience teaching with PhET simulations. All teachers attended a two-hour orientation to teaching with PhET simulations. Over the school year, each teacher taught between 1-3 sim-based lessons, with 13 lessons taught in total. A wide variety of data was collected over the course of the study, including surveys, interviews, classroom video, and content assessments.

Two sources of student data are used for the current analysis. First, after each PhET lesson, students completed a post-PhET reflection about how the experience compared to their business-as-usual lessons. This survey include likert-style questions about enjoyment and learning compared to “other math lessons”, as well as an open-ended prompt asking them to explain why they selected the responses they did. Second, students completed a post-study survey at the end of the study. This survey probed student attitudes about technology and mathematics learning, and included an additional open-ended prompt: “Please describe an activity you found particularly interesting or engaging in math so far this year”. For this study, both open-ended prompts were coded for statements related to the activity, reality, and level principles.

At the end of the study, each teacher was interviewed about their experiences teaching with PhET simulations and how these experiences aligned with their professional goals and perspectives. All interviews were video recorded and transcribed. Post-study interviews were analyzed and coded for statements related to the RME principles, to investigate how these principles were (or were not) apparent in how the teachers discussed their experience.

The coding protocol was similar for all data sources for this analysis. Instances where teachers or students discussed learning experiences that related to one of the RME principles were coded and tallied. Instances were coded for the level principle when teachers discussed progressive formalization (how the simulation helped bridge the informal and the formal) and when they discussed how the simulation provided alternate representations. For the activity principle, instances of student discourse about mathematics and engagement in mathematical activity with the simulation were both coded as part of the activity principle. For the reality principle, instances of the mathematics or their mathematized interactions with the simulations, seaming more relatable, imaginable or grounded in real experiences were coded.

After discussing the coding protocol, raters scored one teacher interview independently (interrater agreement of 83%) and then reflected on similarities and differences to come to a consensus on all instances. Raters also tracked quotes that represented each teacher’s thinking with respect to the RME principles.

Simulations & Lessons: Alignment with RME

In this section, we examine the design of PhET simulations and sim-based lessons through an RME lens, evaluating the alignment of these tools and lessons with RME's activity, reality, and level principles. We use three PhET simulations - *Function Builder*, *Proportion Playground*, and *Expression Exchange* - and associated lessons to contextualize and ground this discussion.

RME in Simulation Design

While the history of the PhET project is rooted in science education, PhET's design principles share much in common with the RME principles. Throughout the design process, the team - guided by both PhET's design principles and the Common Core State Standards - focuses deeply on the context, representations, interactions, feedback, and scaffolding built into the simulation to support students learning experience and teacher facilitation. The resulting math simulations demonstrate significant alignment to the RME principles.

Activity Principle

A central design goal for every PhET simulation is to create a rich, exploratory learning experience, specifically designed to allow and attract students to actively engage with the content (i.e., explore and discover new ideas; practice applying the ideas being learned; and receive feedback on their thinking, allowing them to monitor and correct their own understanding). In *Function Builder* (Figure 1), students build their own functions, choosing how they want to combine or reorder operations. The students drag inputs through and collect corresponding outputs, and, using pattern-based reasoning, students can engage in mathematization as they explore and build their own definition of function as a rule that assigns to each input exactly one output. Similarly, in *Expression Exchange* (Figure 2), students are invited to create collections of coins, exploring which coins can be combined in expressions and which cannot.

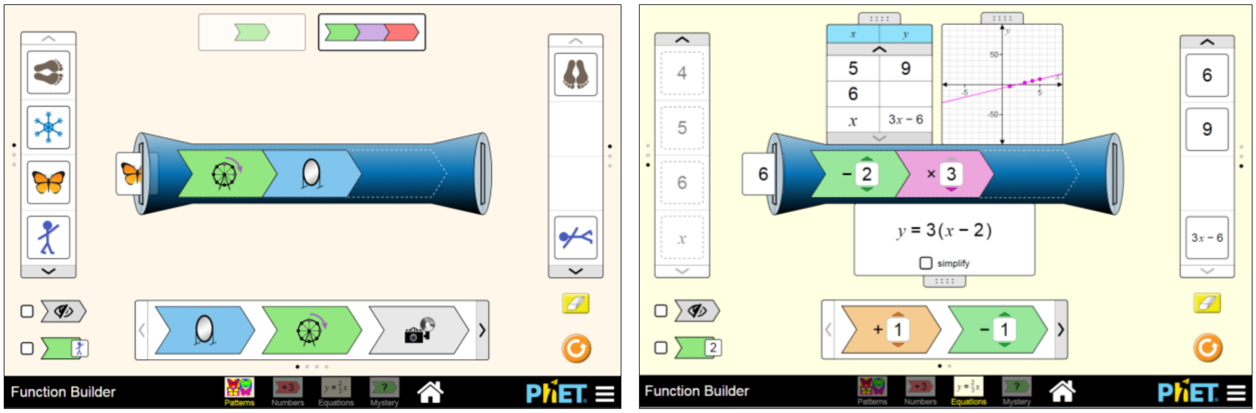

Figure 1: The "Patterns" and "Equations" screens of PhET's Function Builder simulation allow students to create their own functions. (Images by PhET Interactive Simulations, licensed under CC-BY 4.0)

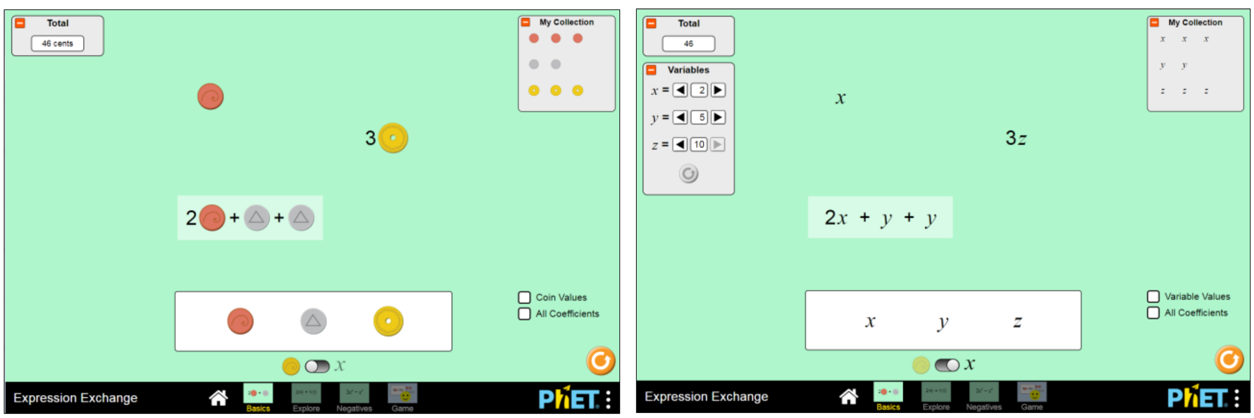

Figure 2: The "Basics" screen of PhET's Expression Exchange simulation allows toggling between the concrete coin representation and formal variable representation. (Images by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY 4.0)

Importantly, there is no "one way" to explore a PhET simulation. The students are in control, and drive their interaction with the sim. They can, and do, ask their own questions (and answer them through sim exploration), configure their own scenarios, make predictions and test their ideas. In the *Proportion Playground* simulation (Figure 3), for instance, students can choose to mix paint, build necklaces, size pool tables, or price apples. Students can easily and dynamically build and compare ratios in any context. Immediate feedback provides opportunities to mathematize in each scenario, e.g. identifying ratios that generate the same necklace pattern or paint color, exploring when mixed paint colors are close or far in shade, or deciphering the unit cost of an apple.

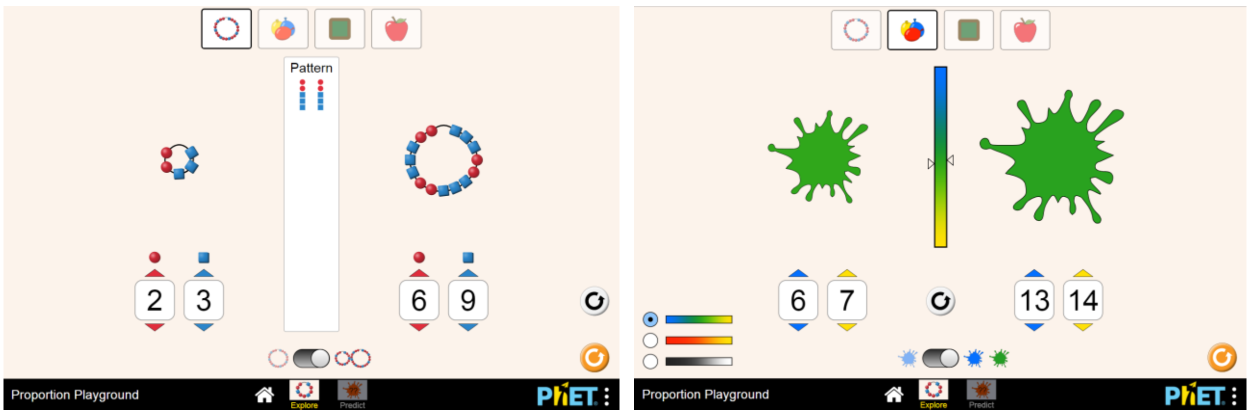

Figure 3: The "Explore" screen of PhET's Expression Exchange simulation provides concrete contexts to explore unit ratios, create equivalent ratios, and make ratio comparisons. (Images by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY 4.0)

Reality Principle

As with many digital tools, the simulations align strongly with the reality principle through their design, creating an environment that can be directly manipulated by the students. The context chosen for the simulation design may tie directly to a real-world situation that embodies mathematical ideas, such as mixing paint or collecting coins. Alternatively, the simulation context may create a concrete and manipulatable representation of symbolic mathematical systems, such as building and simplifying algebraic expressions or ordering and changing functions. Even if the simulation does not have a "real world" context, it is designed in a way that students' interactions with it provide real experiences to help them mathematize. For each simulation, careful attention is paid to the design of the dynamic interaction and immediate feedback cycle, with the goal of encouraging productive exploration and enabling mathematical sensemaking.

In *Expression Exchange*, for example, students first actions are typically to add coins to the play area. As students explore, they discover that they can combine like coins, but not unlike coins. They see coefficients used to help count their coins. They combine their coins into expressions, and discover these expressions can be simplified. With the toggle between "coins" and "*x*", students can now engage in similar exploration and play with the variables *x*, *y*, and *z*, building connections between the concrete collection of coins and that of variables. In this sim, the realistic context of coins serves as a metaphor for mathematical abstractions and strategies.

Level Principle

Significant attention is paid to how a simulation's design scaffolds students' experience and learning. The opening screen of a PhET simulation is explicitly designed to encourage inquiry and discovery of relationships, often using informal contexts before bridging to formal abstractions - such as the coins and variables in *Expression Exchange*.

The *Function Builder* simulation serves as another example. In the first screen, students have the opportunity to develop the informal notion of a function as "an action that can change, or not change, the input" in the concrete context of a set of operations (rotating, flipping, coloring, resizing) that act on pictures. Students can combine and reorder operations, finding that order matters. They can discover that some operations are reversible, while others are not (e.g. making an image black and white). They observe that each input can be mapped to a single output for a given function. In the subsequent screens of *Function Builder*, students build mathematical functions using the addition, subtraction, multiplication, and division operators, and are able to map many of their informal observations to formal mathematical ideas about operations (e.g. the identity function, irreversible functions, order of operations, etc.).

The bridge from informal to formal mathematical ideas is often supported in the simulations through the use of multiple representations. In the *Function Builder - Equations* screen, students can access table, graph, and equation representations of their built function.

These representations change immediately and dynamically with any change the students make in their function. Furthermore, students can not only drag number values through their function, they are also able to drag an “x” card through, adding the representation of a line to the graph and supporting the abstraction from a set of input-outputs to an equation.

RME in Sim-based Lesson Design

For each of the mathematics simulations, lessons were designed to support teacher and student use. All of the lessons were designed with a similar structure that included a period of open play to become acquainted with the objects on the screen and ways in which the objects could interact. This was followed by a whole class discussion of the simulation, and what students observed during the open play session. The lessons then continued with a sequence of prompts and questions that focused on mathematical relationships, and possible conjectures that could be proposed about representations that they created on the screen. These instructional sequences varied in length depending on instructional goals, and the features of the simulation that could be leveraged to address those goals.

Activity Principle

With respect to the activity, reality and level principles, the design of lessons prioritized the open play time allocated at the beginning of the lesson. From prior experience with PhET science simulations, the design team recognized the importance of giving students an opportunity to explore and make sense of various representations and ways they could be moved and changed on the screen. As part of this open play students were also encouraged to explore what switches, check boxes, and carousels revealed and how those changed objects (or not). We would argue that this period of student sense making and interpretation, a form of student-centered engagement, is closely aligned with the activity principle. Students are learning by doing without being given much explicit instruction in advance of open play. Students are also mathematizing objects and relationships, with some of this activity leading to mathematical conjectures. The subsequent whole class discussion and additional prompts builds off of students' ideas, with the lesson sequence focusing on one or more of these concepts and/or skills.

Reality Principle

With respect to the reality principle, since the simulations include concrete objects for students to move, combine, or change, the lessons are organized to support student exploration of these objects, and in some cases specific problem contexts. For example, the *Proportion Playground* simulation includes four problem contexts: String of Beads, Pool Table, Paint Balloons, and Box of Apples. For this research study, the lesson was designed to focus primarily on the Paint Balloons context and the ways in which ratios of two paint colors created different hues. During the open play period, the lesson included the following prompts for teachers to use with students:

- What is true about two the number of blue and yellow used if the color on the right is more blue? More yellow?
- Can you have a color that is more blue, even if it has fewer blue balloons than the other color?
- What is always true about the number of balloons used of each color if the color on the right is the same as the color on the left?

Students first encounter with ratios in through their interaction with paint balloons. Student interaction with the context is the first phase of the lesson, and is used to provide a site for mathematization. The observations are embedded with a context that invites conjectures about similar hues (and ratios of different colored paint), and the similarities and differences in interpreting a visual representation (i.e., combination of paint colors) and a related numerical representation (i.e., ratios). In this way, when appropriate, the lessons were designed to apply the reality principle.

Level Principle

The use of context, models and tools, as well as formal representations, that are built many mathematics simulations was an opportunity to apply the level principle in lesson design. It is important to recognize that the existence of contexts and representations is no guarantee that progressive formalization will be exemplified in the enactment of instructional sequences. Contexts can be underutilized and representations can be used in ways that are disconnected from the related informal and formal mathematics. For example, the *Proportion Playground* activity sheet has students start by using the simulation to compute equivalent colors (ratios), then they discuss more generalizable strategies and eventually they need to compute ratios that go beyond what is allowed by the simulation (too many balloons). Without the activity sheet as a guide, students might not use this simulation to develop more formal understandings of ratio and proportionality. The activity sheet and related lesson helps support the more generalizable and abstract ideas that are established by the simulation.

Analysis of Student and Teacher Data

Quantitative and qualitative analysis of the interview and survey results show that both teachers and students, without being specifically prompted, discuss connections between their classroom experiences with PhET simulations and principles of Realistic Mathematics Education. Teachers make more connections to RME principles than students, and their reflections span a wider and more sophisticated range of principles, although this distribution is not observed for all teachers. Student connections to RME principles are relatively scarce in their survey responses, and when present, are focused almost exclusively on the activity and reality principles.

Quantitative Analysis

In their post-study interviews, most teachers readily made connections to RME principles (Figure 4), with a total of 73 statements coded as related to the activity, realistic, or level principles. We see that the level and activity principle were considerably more common in teacher interviews than the reality principle. We also see variation across teachers. Brandon and Alice (pseudonyms) made fewer connections with the RME principles overall, and while Chloe did talk about PhET in ways that aligned with RME, she only did so for the level and activity principles. These findings suggest that the experience of teaching with PhET simulations was taken up differently across teachers.

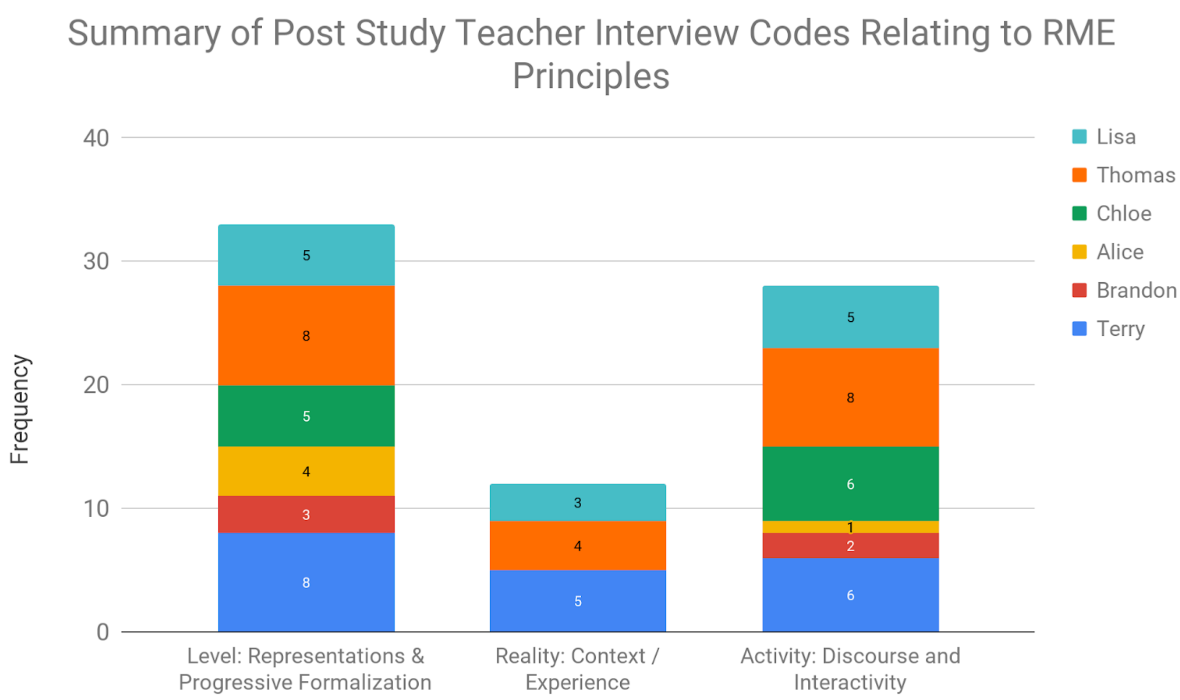

Figure 4: Analysis of teacher post-study interview coding as it relates to RME principles. Teacher names in the legend are pseudonyms

Few students made connections to RME principles in their open reflections about sim-based lessons and their math learning experiences. The coding of student post-PhET lesson reflection responses identified in only 39 out of 388 students responses (or ~10%) with connections to the activity, reality, or level principles (Figure 5). Within these responses, we see a clear pattern that students more often discuss the reality and activity principles, than the level principle. We speculate that this pattern emerges because the pedagogical approaches used to address the reality and activity principles are more tangible to students and their experiences in the classroom - promoting student interactivity, engaging students, and making the content more relevant.

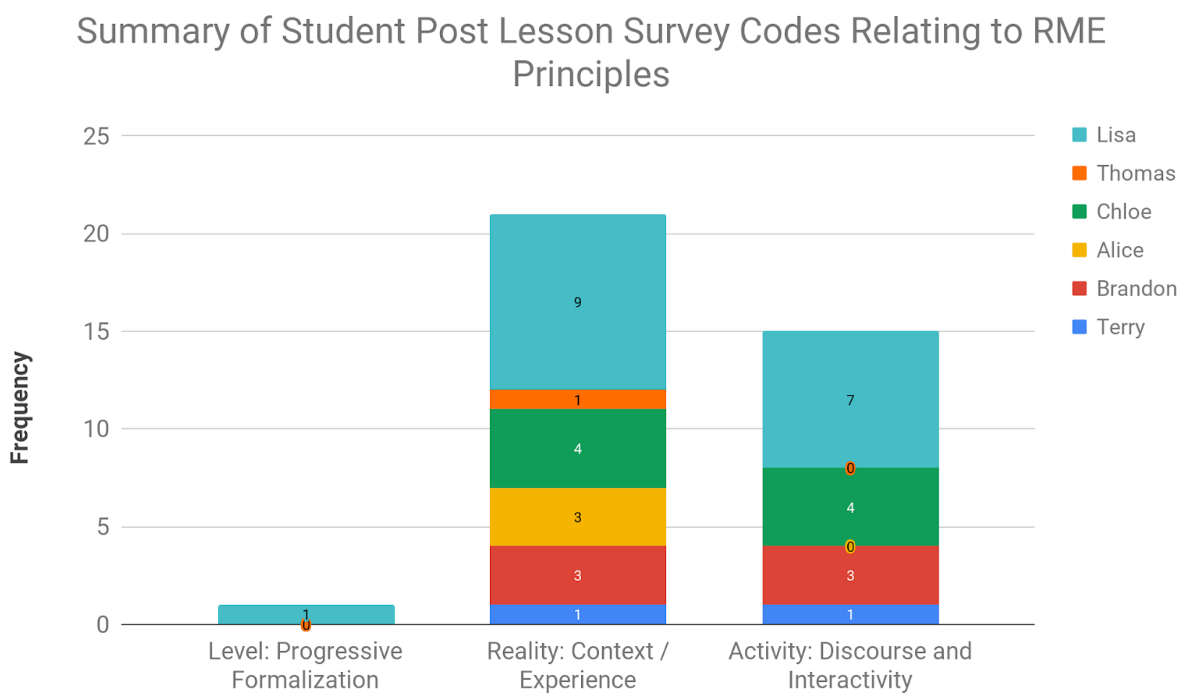

Figure 5: Analysis of student post-lesson reflection coding as it relates to RME principles. Teacher names in the legend are pseudonyms. n=39 out of a total of 388 student responses.

Of the 307 student post-study survey responses, only 13 students chose to discuss PhET simulations when describing an activity they found interesting or engaging in their math class over the year. Of the 13 students, 9 responses were coded as related to the activity principle, 4 to the reality principle and 1 to the level principle (one response was double coded). These findings are consistent with the trend in the post-lesson reflections that students who discuss RME principles, more commonly discuss the activity and reality principles but not the level principle.

Qualitative Analysis

Qualitative analysis comes from coding of teacher post-study interviews and student post-lesson reflections. Analysis is broken down by how responses aligned with the activity, reality and level principles.

Activity

In their post-lesson reflection responses, students talk about enjoying the interactive and collaborative elements of the experience using the PhET simulations. One student says that they “liked this lesson because it was more interactive so it was fun. I understood a bit better because I could see what I was doing more clearly. I learned about the same because I know most of the stuff we are doing.” Although they say they already knew how to do it, they found the interactive component more understandable. Another student says, “The simulator helped me graph out what I was doing so I could go off of it, and use that example to help me. If I got stuck, I could ask a peer, and they would show me, not just tell.” This student explains how in the context of learning with the simulation, it was easier for them to help each other because they could show each other how things work in relation to the simulation, not just tell each other how to do it.

This student’s mention of discourse was not an anomaly, several mention how they found it helpful to have discussions and work collaboratively when learning with the simulations. Another student explains that they liked the simulation activities, "Because being able to explore by ourselves then collaborating with the class helps me understand the lesson and also taking notes." They found the collaborative element of the activity sheets (as well as the exploration component) helpful. A third student states, "I liked this week’s lesson cause we got to do things with an elbow partner. I understood a little bit more. I learned a lot." The peer to peer activity was helpful for them. Finally, a student describes how they got to mediate their thinking by talking with peers in a group. "The reason why I liked it more because it was something was more writing down on paper. I understood it more from it being a group activities, less than one on one. I learned more a lot when others asked questions." The interactive and collaborative components of the simulation activities were particularly popular for the students.

Teachers also talk about the benefits related to the activity principle. They focus on the discovery component of the simulation activities as well as how students learned through exploration and play. Lisa explains,

> With PhET, you know it's not quite that type of exploration or fun play but they were really discovering and you know and at first I thought: oh they're never going to figure out what this button is on the *function builder*. They're not going to figure out that you can see it go, I think the eyeball, and they figure it out. Yeah. And then they get excited when they figure it out rather than being told. So I would tell future teachers definitely let them explore and ask them: teach me. Tell me. You know even though you do know what's going on, they are so excited to be able to share their knowledge of something that they just discovered on their own. They're very proud of that. (Lisa, personal communication 2018)

She found that the students really liked figuring out things on their own and then explaining what they found. These connections to making a more active mathematics classroom also aligned with teachers professional goals.

> So the PhET is better in that, they get to explore. I mean, a highlight I like that they get to explore and they get to play with it and I'm just more facilitating, which is one of the goals, I'm trying to get more to that as a teacher (Alice, personal communication 2018)

Students were exploring more and discovering mathematical concepts along the way. This fit well with how some of the teachers want their students to learn. Thomas said that in addition to noticing this type of interactivity, it aligned well with his goals.

> I think anytime students can discover something themselves rather than being told it, it increases their retention of it and also their interest in it. When students discover a pattern for themselves they kind of have a sense of pride like; Hey I figured this out on my own, and I think things that students take pride in they're more likely to perform on. And so in that way I think it was beneficial to the kids that did discover things (Thomas, personal communication 2018)

He finds that his students have increased ownership over their learning when they can discover things, improving motivation as well as comprehension.

Teachers, like their students, noted the benefits of students talking to each other and having on-task conversations. Chloe discusses in her post interview how students in the PhET group “were collaborating a lot, playing with the app, asking questions that they probably wouldn't have thought of without the simulation, which was nice. (Chloe, personal communication 2018). By getting her students to talk to each other about the simulation, she has them engaging as active participants in their learning. Chloe was not alone in this type of thinking, Terry thought that the conversations students had during these activities were productive.

> I think I like it because they were able to see different things and try different things and then when we shared out I liked the conversations we had after. When we would like come back and everybody is like; oh I saw this and I saw that. And if you switched this toggle to this toggle, it changed your vision or your view of what's happening. (Terry, personal communication 2018)

By talking about the mathematics, students can collaborate and work to co-construct meaning with the teacher as a guide. This starts to get into other RME principles (such as guided reinvention) although it would not be possible if not for the interaction between the students.

Reality

Students in both the post-lesson reflections (21 responses) and the post-study survey (4 responses) talk about how they thought the PhET simulation experience made the mathematics more real to them, more understandable, and more relatable. One student said, "I liked it because I could see what a function actually does and not just write it down." The student enjoyed the *Function Builder* simulation lesson because it allowed them to see the actual meaning behind what was happening mathematically, not just memorize the procedure. Another student writes, “I liked the graphing simulation because although it was harder to use, it helped me understand how to use graphing. It helped me understand the graphing in real world situations”. This student is drawing connections between the mathematical activity to their lives outside of the classroom.

A number of students commented about being "visual learners", and benefits of the visual representations present in the PhET simulations. One student wrote, "It was more visual and simpler to understand". Another wrote "I like this weeks lessons because I understand it and it is easier to do. I also learned more because I am a visual learner." These comments suggest the simulations aided these students by helping them to visualize what was happening, making it more real to them than abstract mathematics where intermediate steps or alterations of parameters are left unseen.

Student responses were not universally positive, however, with 2 students who didn't find the experience helpful in making the mathematics more real. One students said, "It was somewhat fun to do but I didn't understand it. The pictures just confused me and I didn't get how it related. With the numbers I understood." They did not find the pictures in the *Function Builder* helpful in making the mathematics more imaginable and real to them. The other student said "I didn't really learn as much. The positive though is the visual representation."

Teachers also discussed how they liked that the simulation lessons made the learning feel more real to their students. Thomas talks about how his students found the simulations more approachable, he says,

> Kids that are really stressed out by looking at numbers and trying to see the relationship between numbers were a lot less stressed looking at you know a butterfly that rotated 90 degrees. That was a lot more approachable in its nature than saying; how do you turn the number three into the number negative 30. I think they were able to find those connections a lot better visually rather than looking at numbers. (Thomas, personal communication 2018)

He thinks that the visual representations in the *Function Builder* lesson made the connections more tangible and approachable for his students, better allowing them to let the math be real to them. Lisa also thought that the PhET simulations reached students on both ends of the achievement spectrum, allowing them all to better see the purpose behind what they were doing.

> I feel like the PhET reaches a more broad spectrum of students. You know, the higher end kids who already get math. They have a better richer understanding because a lot of

> them can just follow procedures but don't understand the why behind it. But then the struggling learner can actually comprehend what's going on because they can see the math and it's not just a bunch of... A recipe you follow. They see purpose behind it. They see outcome.

She thinks that the simulations made the procedures of the mathematics appear more accessible and relatable, helping her students better engage with the content. We see that teachers, as well as students, found the simulation activities made the mathematics more realistic and approachable through the visual representations.

Level

Analysis of the student data showed students making very few connections to the level principle - with only 4 coded responses across all student post-lesson reflections and post-study surveys. In addition, these few responses showed variation across students. Two students commented about the helpfulness of simulations in supporting more formal math ideas. For instance, a student wrote in their post-study survey, “I liked the *function builder*. I liked that simulation because it allowed me to explore with a mathematical idea without using math for the first part of it. Then the teacher was able to teach the skill without spending too much time on the basics”. This student was “interested” and ”engaged” by exploring the mathematical idea of functions in a pre-formal context before learning it more formally. Two other students, however, commented that the simulations unhelpful in ways related to the level principle. One commented “I learned about the same because I did not understand how the PhET simulation would be easily been applied in larger more complex problems.” For this student, the bridge between the simulation and more complex, abstract mathematics was not clear to them. Overall, the shear sparsity of student reflections concerning the level principle suggest that the connections are less observable to students than the connections to the other RME principles.

Unlike their students, the teachers in the study consistently drew connections to the level principle in their reflections on the experience of teaching with PhET simulations. Some of the teachers reflections were quite explicit in their connections to progressive formalization. Thomas stated,

> I think connecting math theory with like reality or connecting visual representations of math with abstract relations of math or abstract representations, I should say of math. Connecting the abstract and the concrete was a lot better in the PhET simulation. In my normal lessons I typically jump right into the math right away you know, I'll do things like the function machine initially and then I'll jump right into kind of the more abstract. With the PhET simulations, we spend a lot more time working with concrete things that they can see and manipulate before we turn it into something like equations solving. And so I think that's good, is the connecting the concrete with the abstract, was better in the PhET simulations than in a typical class. (Thomas, personal communication 2018)

He found the PhET simulation lesson helpful in getting his students to draw connections between more relatable, less formal notions of mathematics with formal, abstract math. He later elaborates further with a specific example from one of the lessons,

> I think that particularly the *Function Builder* lesson was good because I think when kids see y=3x+7 they don't really understand what it means. But when they see a machine where something goes in and something comes out now they really understand input and output rather, than just X and Y. So I think when bridging the gap between the concrete and the abstract, the function machine was really helpful for doing that. Because a lot of kids, when you show them an equation that involves variables, they don't really understand abstractly what a variable means, how to work with variables, and things like that, but everyone understands that something goes in and something comes out of a machine and that really put function vocabulary in a good context so that when we did talk about X and Y they could understand that; OK X is something that goes and Y is something that comes out. The function machine is how to turn that X into Y. And I think that gave them a very good framework for understanding what the functions were themselves. (Thomas, personal communication 2018)

This teacher describes how he thinks the function machine in the *function builder* lesson helps students draw connections between the idea of concrete inputs and outputs and abstract functions with Xs and Ys. He says that it is helpful to be able to discuss the simulation and leverage the informal concepts in the simulation.

He was not alone in his descriptions of how the simulation could help students learn in progressively more formal levels of understanding. In his post-study interview, Brandon stated that,

> I think [PhET] was another, like another crutch for them to use, the lower, the lower level kids, instead of seeing it the way I do it or the way [the commercial, business as usual curriculum] does it. But having a third visual perspective to match it up instead of getting hung up on letters and variables and math they got to see it with circles and squares and then easily change that. It was a smooth transition just to change. (Brandon, personal communication 2018)

This teacher talks about pre-formal representations as a "crutch" for lower achieving students which, despite the negative connotations, can be interpreted as the level principle. He notes that his students can easily change from the pictures to the variables and that it supports their mathematical development.

The contrast between student and teacher responses is notable. All teachers discussed the level principle and thought that the simulation helped their students formalize their understandings. Opportunities for students to more vividly experience the connection to and benefits from the level principle afforded by simulations might be improved with more explicit attention to building clear connections between the simulation activity and the formal mathematics within each sim-based lesson. Opportunities for students to reflect on how they learn and why they are using the simulation could be helpful in bridging the simulation with more formal mathematical ideas.

Summary and Conclusions

Simulation-based lessons were designed to elicit mathematical activity that is aligned with the principles of RME. Our analysis of the simulations and the sim-based lessons shows ample evidence of alignment with the activity, reality, and level principles of RME. The simulations often provide a relevant and pre-formal representation of the mathematics concepts. The simulations also afford direct student interaction and manipulation with multiple dynamic representations and immediate feedback. The lessons and activity sheets leverage these affordances and combine them with pedagogical support for opportunities to progressively formalize mathematical understandings and to learn through productive discourse between students.

Analysis of data collected after implementation of these sim-based lessons in 6th-8th grade classrooms shows how the student and teacher reflections on their actual experiences relate to the RME principles. Connections to the activity principle were made by both students and teachers. Students discuss enjoying learning with the simulations because they were interactive and supported peer-to-peer interactions that they found valuable. Teachers talk positively about the simulation and lesson affordances for discovery and productive play, aligning these concepts with their professional goals. Students mentioned how they appreciated the tangible and accessible nature of the simulation, aligning with the reality principle. Only half of the teachers talked about the simulation lesson helping students by making the content more imaginable and by placing it in a realistic context. And those teachers who did make connections to the reality principle, discussed how this was beneficial for differentiation and increasing motivation of students they deemed ‘lower level’. Teachers frequently mentioned how they enjoyed supports for progressive formalization and dynamic representations as virtual manipulatives. They thought these components of the experience, aligned with the level principle, were helpful for their students. Similar reflections were not apparent in the students; two students expressed confusion and frustration about the lack of connections between the sim-based activity and formal mathematics.

These findings are significant for designers and educators interested in teaching with digital interactive simulations. Simulation designers can support teachers in making simulations that afford opportunities for active learning, progressive formalization, discovery, and ways for students to leverage realistic contexts in mathematical discourse with each other. Classroom activities that use simulations can also be effective at supporting productive discourse about mathematics, using the simulation as a powerful mediator of content and opportunity for students to use evidence in their arguments. More instructional design attention should be paid to helping support connections between the informal and pre-formal concepts in the simulation and the formal mathematics that it is being used to teach. Having students draw connections between levels of formality is crucial to the success of any simulation-based lesson.

Acknowledgements

The authors extend our thanks and gratitude to the students and teacher participants and to the entire PhET team for their support of this work. The research reported here was supported, in part, by funding from the National Science Foundation under Grant No. DRL-1503510. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the NSF.

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