TEACHER ATTRIBUTES AND SCHOOL CONTEXT: WHAT ARE THE BARRIERS TO DISCOURSE IN ELEMENTARY MATHEMATICS?

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This study utilized a multilevel model to examine the relationship between teacher attributes, school context, and the characteristics of the discourse within novice elementary mathematics lessons. MKT and teacher beliefs were significant predictors of the level of student explanation within teachers' lessons while school SES and perceived levels of support accounted for variance in the overall mathematics discourse community.

Keywords: Classroom Discourse, Elementary School Education, Mathematical Knowledge for Teaching, Teacher Beliefs

Introduction

Research has shown that elementary teachers are often anxious about teaching mathematics due to their prior experiences (Stevenson & Stigler, 1994), hence the work of teacher preparation programs to reverse this trend among their preservice teachers (PTs). Preparation programs strive to build PTs' confidence in teaching mathematics by improving their specialized content and pedagogical knowledge. One element of mathematics teaching and learning on which teacher preparation programs typically focus is the use of mathematical discourse to support students' justification and explanation of ideas. The importance of mathematical discourse has been supported by research (e.g., Michaels & O'Connor, 2015), emphasized in standards (National Governors Association Center for Best Practices, Council of Chief State Officers [NGA & CCSSO], 2010; National Council of Teachers of Mathematics [NCTM], 2000), and echoed in the recently published *Principles to Actions* (NCTM, 2014).

Despite the emphasis on the importance of discourse, research has shown that high-quality mathematical discourse is not the norm in U.S. classrooms (Michaels & O'Connor, 2015); therefore, more research is needed to understand the borders that promote and hinder its actualization in elementary classrooms, particularly for novice teachers who recently graduated from programs advocating its use. This study aims to fill that void by examining the relationships among teacher attributes, school contextual factors, and mathematical discourse. By studying novice teachers, we attempt to investigate the bridge between teacher preparation and induction into the profession and the borders that are productive or problematic for the implementation of high-quality discourse.

Theoretical Perspective

To study the co-construction of discourse within the mathematics classroom, this study employs a situative approach to learning (Lave & Wenger, 1991; Cobb & Yackel, 1996). A situated perspective allows one to conceptualize the teacher as an influential "old timer" in the classroom community of practice where students learn the expected norms through their interactions with the teacher and fellow classmates (Lave & Wenger, 1991, p. 29). Further, broader school contextual factors are often considered when attempting to fully situate learning.

This study attempts to gain insight into the mathematical communities established in novice teachers' classrooms by examining the classroom using various units of analysis. We first focus on the novice teacher's "distinct way of being" (Skott, 2009, p. 31) by attending to attributes measured at the individual level. Next, we explore the school context in terms of teachers' perception of support and school demographics, which shed light on the sociocultural influences. Lastly, using a participatory lens, we explore the nature of the classroom community of practice through an

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examination of the type of discourse that occurs within mathematics lessons. As a result of various units of analysis within the community of practice, we are able to explore associations between teacher attributes, context, and mathematical discourse.

Related Research

Discourse

From a situative perspective, communication is essential to the development of mathematical concepts and therefore is a necessary component of quality mathematics instruction (Cobb & Bowers, 1999). If students are to engage in authentic practices of mathematicians, then they need opportunities to discuss mathematical ideas and justify their reasoning (Lave & Wenger, 1991).

When communicating ideas, students challenge and reinforce the learning within the classroom (Sfard, 2006). In order for this co-construction of knowledge to be possible, students must feel comfortable and supported in their efforts to dialogue with one another and the teacher. The sociomathematical norms are developed through interactions between the teacher and the students (Cobb & Yackel, 1996). The types of questions posed, opportunities for student-to-student discourse, and solicitation of students' ideas are ways that the teacher explicitly and implicitly sets the norms and builds the discourse community.

In addition to the overall participation of students in the community discourse, the level to which students explain and justify their reasoning depends upon the teacher's actions. As teachers press for further explanation, they can simultaneously model the "sophistication" expected within mathematical responses (Cobb & Yackel, 1996). Michaels, O'Connor, and Resnick (2008) state that teachers support "accountable talk" by holding students accountable to their reasoning and knowledge.

Teacher Attributes

The teacher is influential in the classroom community of practice (Lave & Wenger, 1991) by drawing upon his/her knowledge and beliefs when making instructional decisions, and in turn cultivating the community norms. First, teachers need specialized content and pedagogical knowledge, often referred to as mathematical knowledge for teaching (MKT), to effectively facilitate mathematics learning (Ball & Bass, 2003). Research has shown that teachers' MKT influences the use of standards-based instructional practices (Hill et al., 2008) and student achievement (Hill, Rowan, & Ball, 2005). This study takes a closer look at the discourse within the community of practice to better understand the relationship to teachers' MKT.

In addition to knowledge, research has indicated that teachers' instructional practices are related to their beliefs about mathematics (Stipek, Givvin, Salmon, & MacGyvers, 2001), beliefs about teaching and learning (Walshaw & Anthony, 2008), and beliefs about their own ability to teach (Beard, Hoy, &Woolfolk Hoy, 2010). The mathematics education field continues to sort out definitions within the beliefs literature (Philipp, 2002), but this study addresses teachers' epistemological beliefs (beliefs about the nature of mathematics and learning) and personal mathematics teaching efficacy (PMTE; Enoch, Smith & Huinker, 2000).

School Context

The school culture in which a classroom community of practice is situated has impacts on actualization of the teachers' goals for his/her classroom (Skott, 2009). Teachers specifically cite administrative and team-level support as influential in their instructional choices (Cobb, McClain, de Silva Lamberg, & Dean, 2003). This study accounts for teachers' feelings of support, specifically in regards to mathematics instruction.

Well documented within research is the relationship between socioeconomic status (SES) and students' mathematics achievement (e.g., Ma & Klinger, 2000). However, studies analyzing the associations between mathematics instruction and SES are often using broad categories such as procedural or conceptual instruction; there has been little attention to the interactions within the classroom. This study examines the discourse community and level of explanation and justification and their relationships to the overall SES of students.

Methodology

Participants

Participants in this study were 118 novice elementary teachers in their second year of teaching. Participants, with an average age of 23, taught in various elementary schools across one southeastern state and also completed their undergraduate teacher preparation program at public universities within the state. Propensity score matching (Fan & Nowell, 2011) was used to create a comparable sample based on college entrance characteristics including SAT/ACT and high school grade point averages. From this sampling technique, teachers were identified and recruited to participate. Overall, the sample was 98% Female and 85% Caucasian which is representative of alumni of the preparation programs. Descriptive analysis of school level information showed that 70% of teachers in the sample taught at schools that were classified as Title 1 schools and had over 50% of student receiving free and reduced price lunch.

The participants attended a one-day summer session before their second year of teaching. The session included: training on how to use an instructional log (not a focus of this study); training on how to effectively video record instruction; and data collection on a variety of surveys and assessments. The surveys and assessments utilized in this study will be described in the measures section. The training on video recording prepared the participants for recording three mathematics lessons (a focus of the current study) across their second year of teaching.

Nine teachers submitted 1 video, 18 teachers submitted two lessons, and 89 teachers submitted three or more lessons. Teachers with less than three lessons were retained in the analysis because past research using the same observational measure found that less than one percent of the variance in scores on the M-Scan measure (described below) was attributable to the number of observations (Walkowiak, Berry, Meyer, Rimm-Kaufman, & McCracken, 2014).

Measures

The level of Mathematics Discourse Community (MDC) and level of Explanation/Justification (EJ) are the two outcome variables. Dependent variables include teacher-level attributes related to knowledge and beliefs: MKT, PMTE, and EB. Also, school contextual variables are dependent variables and include school SES (percent free and reduced price lunch; FRPL), grade level, and perceptions of the school support.

Mathematics Scan (M-Scan) Observational Measure. The M-Scan is an observational protocol that measures the presence and extent of standards-based mathematics teaching practices (Berry et al., 2010). Of the measure's ten dimensions, this study specifically examines two: Mathematics Discourse Community (MDC) and Explanation and Justification (EJ). The indicators for MDC are teacher's role in discourse, sense of mathematics community through student talk, and questions. The indicators for EJ include presence of explanation/justification and depth of explanation/justification. A trained observer rates a mathematics lessons on these dimensions using a 7-point scale that is divided into three levels: low (1-2); medium (3-5); and high (6-7). There are specific descriptions for each level that are thoroughly defined in the coding protocol. The M-Scan development team gathered sources of evidence of validity and score reliability (Walkowiak et al., 2014).

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Mathematical Knowledge for Teaching (MKT) Measure in Number and Operation (N&O).

The MKT-N&O measure consists of 26 items designed to measure teachers' mathematical knowledge for teaching number and operations (Hill, Schilling, & Ball, 2004). This particular form of the MKT was selected because a majority of K-5 content is focused on number and operations. Item response theory models (IRT) were used with the data and the IRT reliabilities for the different domains of the MKT-N&O were good to excellent, ranging from .71 to .84. Participants completed the MKT-N&O via an online interface; they were allowed to use paper and pencil. Scores were recorded in a master database as an IRT score relative to the national sample.

Mathematics Teaching Efficacy Beliefs Instrument (MTEBI). The PMTE subscale of the MTEBI measure (Enochs, Smith, & Huinker, 2000) is comprised of 13 items and uses a 5- point Likert scale (strongly disagree, disagree, uncertain, agree, and strongly agree) with "strongly agree" denoted by a 5. The PMTE subscale measures a teacher's belief in his or her own ability to effectively teach mathematics. In the developers' work, reliability analysis produced an alpha coefficient of 0.88 (N = 324) (Enochs, Smith, & Huinker, 2000).

Mathematics Experiences and Conceptions Surveys (MECS). The MECS (Jong, Hodges, Royal, & Welder, 2015) was designed to measure pre-service and novice teachers' attitudes, beliefs, and dispositions toward mathematics teaching and learning. This study utilizes the beliefs subscale from the MECS-Y1 version, which is designed for teachers in their first three years of teaching. Validation work (Jong et al., 2015) reported relatively high reliability of this subscale (α = .78). To provide a more precise categorization of epistemological beliefs (EB), an exploratory factor analysis was conducted and revealed three factors within the beliefs domain, which are classified as epistemological beliefs about the nature of mathematics (Nature, α = .60), epistemological beliefs about the use of calculations (Calculations, α = .70), and epistemic value of mathematics (Value, α = .81). These three subgroups are used in the analysis.

Teachers' perceptions of school level support for mathematics instruction (TPSS).Participants also responded to 6 items that gauged their perceptions of barriers within their school to quality mathematics using the *scale significant problem*, *somewhat of a problem*, or *not a significant problem*. An index of TPSS (possible range 6-18) was created from these items to provide a measure of the level of support the teacher felt in regards to mathematics instruction.

School contextual variables. A measure of the school's SES was created using the percentage of students receiving free or reduced price lunch (FRPL). The grade level for each teacher was recoded to create a grade band variable. Kindergarten, first, and second grades were considered primary grades, while third, fourth, and fifth grades were considered intermediate grades.

Analysis

The first step in the quantitative analyses was to measure the discourse present in each lesson using the M-Scan observational measure. A team of five coders was trained on the M-Scan observational protocol during a 12-hour training facilitated by a co-developer of the measure. Each coder established 80% within-one in comparison to the master coder on each of the 10 dimensions. The team met every other week for drift check meetings.

Next, to allow for more meaningful interpretation of results, z-scores were calculated for participants' MKT scores. Other dependent variables including PMTE, EB subscales, FRPL, and TPSS were grand-mean centered to provide a meaningful zero. That is, zero for each variable is interpreted as the sample mean and allows results to be interpreted in reference to this sample.

Central to the study, a series of multilevel means-as-outcomes analyses were performed using SAS software to examine the proportion of variance in the classroom discourse dimensions, MDC and EJ, that is explained by teacher attributes and school context. By examining the variability in MDC and EJ across multiple lessons, multilevel modeling (MLM) is a powerful and flexible approach compared to techniques, such as multiple regression, because estimates of both between-

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person and within-person variability are possible (Lee & Bryk, 1989). Additionally, multilevel modeling uses all available data from each participant and can effectively manage unequal data (Raudenbush & Bryk, 2002).

MDC and EJ were entered into the model as Level-1 variables. Each teacher's PMTE score, EB subscales, and MKT score were entered as Level-2 variables because they describe attributes of the teacher. School contextual variables, FRPL, TPSS, and grade level, were entered as Level-2 variables due to the fact that TPSS represents teachers' perceptions of their school support, grade-level is viewed a classroom characteristic, and school level variables are associated with teacher due to the fact that teachers are not at the same school.

A fully unconditional model/null model (Raudenbush & Bryk, 2002) was conducted to determine the variability within teachers (Level 1) and the variability between teachers (Level 2) on the variables of MDC and EJ. The null model for each dependent variable, MDC and EJ, would serve as the reference to explain variation within the sample. This model was conducted to ensure that there was sufficient variability at Level 2 to warrant continuation with analyses.

Next, a sequential Means-as-outcomes Regression was conducted to test the main effect of predictors at Level 2 (i.e., MKT, PMTE, EB subscales, and TPSS) on teachers' MDC and EJ, separately. First MKT was entered into the model, followed by teacher attributes (PMTE, EB subscales), and lastly school contextual factors (FRPL, TPSS, grade level). For each model, significant main effects, significant interactions, and percentage of variance explained between teachers were examined.

Results

Results from the full unconditional or null model indicated that 37% of the variability in the level of MDC was between teacher (τ_{00} = .51, z=4.57, p<.001) and 63% was within lessons for an individual teacher (σ^2 =.88, z=10.50, p<.001). The fully unconditional model for EJ indicated that 33% of the variability in the level of EJ was between teacher (τ_{00} = .75, z=4.25, p<.001) and 67% was within lessons for an individual teacher (σ^2 =1.51, σ =10.46, σ =1.51, σ =1.51,

Mathematical Discourse Community (MDC)

Table 1 displays the results of the multilevel analyses for MDC. When MKT was entered into the model by itself (Model 2), it was marginally significant (p=.06). However, when controlling for efficacy and the EB subscales, MKT was not a significant predictor of the level of MDC (Model 3). The Nature subscale was a significant predictor of the level of MDC (p<.05). That is, teachers with more sophisticated/standards-based beliefs about the nature of mathematics also tended to have higher levels of MDC within their lessons on average, when controlling for levels of MKT and PMTE. For each scale point increase in the *Nature* score, the MDC score on average would increase .08 points. In comparison to the null model, Model 3 accounted for 24% of 37% variance between teachers.

In the next step (Model 4), school context variables were added. When controlling for all other variables, $Nature\ (p<.05)$, FRPL (p<.05), and TPSS (p<.05) accounted for a significant amount of the variance between teachers in their scores on MDC. For each unit increase in the Nature score, one would expect the average MDC score to be .13 units higher while for each unit increase in the FRPL percentage, one would expect an average decrease of .80 units on MDC. Finally, for each unit increase in TPSS, the expectation is a .06 unit increase in MDC. Overall, Model 4 accounts for 24% of 37% variance between teachers on their MDC scores.

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Math	Wathematical Discourse Community						
	Model 1 DF=118	Model 2 DF=115	Model 3 DF=109	Model 4 DF=100			
Fixed Effects, MDC, β ₀	(SE)	(SE)	(SE)	(SE)			
Intercept, γ ₀₀	3.78**(.08)	3.79**(.08)	3.79**(.08)	3.79**(.11)			
Teacher Attributes							
MKT, γ ₀₁		0.16 (.08)	.11 (.09)	.14 (.09)			
PMTE, γ ₀₂			001 (.02)	.01 (.02)			
Nature, γ ₀₃			.08* (.04)	.13* (.04)			
Calculations, γ_{04}			.03 (.05)	.03 (.04)			
Value, γ ₀₅			.03 (.05)	.08 (.05)			
Teaching Context							
FRPL, γ ₀₆				80* (.38)			
Grade band, γ ₀₇				01 (.18)			
TPSS, γ_{08}				.06* (.03)			
Random Effects							
MDC (τ ₀₀)	.51**(.11)	.48**(.11)	.47**(.11)	.38** (.10)			
Within-teacher variation (σ^2)	.88** (.08)	.89** (.09)	.89** (.09)	.89**(.09)			

Table 1: Estimated Effects of Teacher Attributes and School Context on the Level of Mathematical Discourse Community

Explanation & Justification (EJ)

Subsequently, models were fit to examine the relationships between all dependent variables and EJ (see Table 2). MKT was entered into the model alone (Model 2), and it was a significant predictor of EJ (p<.05). In the next step, controlling for PMTE and the EB subscales, MKT remained a significant predictor of the level of EJ (p<.05), and the *Nature* subscale was significant as well (p<.05). That is, for each unit increase in MKT score, EJ was expected to increase .22 units. Also, teachers with more sophisticated/standards-based beliefs about the nature of mathematics tended to have higher levels of EJ within their lessons, and for each scale point increase in the Nature score, the EJ score on average would increase .13 points. In comparison to the null model, this model accounted for 14% of 33% variance between teachers on their EJ scores.

In the final step, school context variables were added. When controlling for all other variables, MKT (p<.05), and *Nature* (p<.05) remained significant predictors of EJ, while no school contextual variables were significantly related to the level of EJ. When controlling for the other variables, for each unit increase in a teacher's MKT score, one would expect the average EJ score to be .24 units higher. For each unit increase in the *Nature* score, an increase of .17 on EJ is expected. This model accounted for 23% of the variance between teachers on EJ scores.

Discussion

The purpose of this study was to examine teacher attributes and school context in regard to their relationship to the overall mathematical discourse community and the level of student explanation and justification in novice teachers' mathematics lessons. MKT had a significant positive relationship with the level of EJ in a teacher's lesson, but did not impact the MDC. That is, a teacher's level of MKT does not seem to influence their likelihood to solicit student ideas or allow opportunities for student-to-student talk, but it does impact the level of questioning posed to promote students' explanation of ideas. This aligns with previous studies that have shown that teachers with higher MKT are more likely to respond to students' thinking and provide rich opportunities with the mathematics (Hill et al., 2008). This finding supports programmatic efforts within teacher preparation programs that strive to increase PTs' MKT.

^{**}*p* < .001, **p* < .05

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Table 2: Estimated Effects of Teacher Attributes and School Context on the Level of

Explanation and Justification

Explanation and Justification						
Model 1 DF=118	Model 2 DF=115	Model 3 DF=109	Model 4 DF=101			
(SE)	(SE)	(SE)	(SE)			
3.19**(.11)	3.20**(.10)	3.21**(.10)	3.16**(.14)			
	.27* (.10)	.22*(.11)	.24* (.15)			
		01 (.02)	004 (.02)			
		.13* (.05)	.17* (.05)			
		.04 (.06)	.05 (.05)			
		01 (.06)	.05 (.06)			
			76 (.48)			
			.09 (.04)			
			.07 (.04)			
.75** (.18)	.70** (.17)	.65** (.17)	.58**(.17)			
1.51** (.14)	1.50**(.15)	1.53**(.15)	1.50** (.15)			
	Model 1 DF=118 (SE) 3.19** (.11)	Model 1 DF=118 (SE) (SE) 3.19**(.11) 3.20**(.10) .27* (.10)	Model 1 DF=118 Model 2 DF=115 Model 3 DF=109 (SE) (SE) (SE) 3.19**(.11) 3.20**(.10) 3.21**(.10) .27* (.10) .22*(.11) 01 (.02) .13* (.05) .04 (.06) 01 (.06) .75** (.18) .70** (.17) .65** (.17)			

^{**}*p* < .001, **p* < .05

The field continues to diverge in their conceptions of epistemological beliefs. This study teases apart distinct elements of epistemological beliefs and found that teachers' beliefs related to the nature of mathematics were predictive of the MDC and the level of EJ within their classrooms. It seems plausible that a teacher must believe that mathematics is an interactive discipline before employing discourse-based teaching strategies. This finding confirms the importance of attending to and explicitly addressing PTs' beliefs because beliefs may be a border preventing some teachers from constructing opportunities for classroom discourse.

School contextual variables, FRPL and TPSS, also accounted for differences between teachers' MDC. Teachers within schools with higher FRPL had lower levels of MDC, meaning that on average students of low SES had less opportunity for sharing their ideas with their teacher and fellow classmates. This finding supports previous research that students of SES receive more procedural instruction (Desimone & Long, 2010) in that procedural instruction is less likely to include discussions of ideas. Also, teachers reporting higher levels of school support in regards to mathematics instruction were also more likely to solicit student ideas and allow time for students to discuss mathematical ideas with one another. This positive relationship between the school culture and classroom practice has implications for administrators and instructional support personnel (e.g., coaches) as they work to promote quality instruction. Messages need to align between the mathematics education field and the school culture to advance discourse-rich classrooms. School context is influential; next steps for this work will include a more in-depth examination of the discussions in classrooms of a variety of SES. SES should not be a border that keeps some students out of mathematical conversations because it is those conversations that provide access to the mathematics.

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