International Journal of Mathematical Education in Science and Technology

ISSN: 0020-739X (Print) 1464-5211 (Online) Journal homepage: http://www.tandfonline.com/loi/tmes20

Variations on play with interactive computer simulations: balancing competing priorities

Ian Whitacre, Karina Hensberry, Jennifer Schellinger & Kelly Findley

INTERNATIONAL JOURNAL OF MATHEMATICAL EDUCATION IN SCIENCE AND TECHNOLOGY
https://doi.org/10.1080/0020739X.2018.1532536

Taylor & Francis
Taylor & Francis Group

Check for updates

Variations on play with interactive computer simulations: balancing competing priorities

Ian Whitacre[a], Karina Hensberry[b], Jennifer Schellinger[a] and Kelly Findley[a]

[a]School of Teacher Education, Florida State University, FL, USA; [b]College of Education, University of South Florida St. Petersburg, FL, USA

ABSTRACT

U.S. mathematics teachers face considerable pressures to keep up with pacing guides and to prepare students for standardized tests. At the same time, they are called upon to engage students in innovative exploratory activities and to incorporate new technologies into their lessons. These competing priorities pose considerable challenges. Against this backdrop, we investigated how middle-school mathematics teachers incorporated play into lessons involving interactive computer simulations (sims). The teachers used PhET sims in a variety of lessons. Following general guidelines for teaching with PhET sims, these lessons included a short period of play prior to more structured work with the sim. Our analysis of 15 mathematics lessons involving play led to the identification of four characteristics that distinguish the play phases of these lessons. Based on combinations of these characteristics, we identified three specific profiles of play, which lie at different points along a continuum of priorities from foregrounding students' ideas to keeping pace. We discuss the implications associated with each profile of the play phase, and we begin to articulate a theory that frames teaching with play as a matter of balancing divergent and convergent modes of activity.

ARTICLE HISTORY
Received 14 July 2017

KEYWORDS
Interactive computer simulations; play; demands on teachers

> If you once let children evolve their own learning along paths of their choosing, you then must see it through and maintain the individuality of their work. You cannot begin that way and then say, in effect, "That was only a teaser," thus using your adult authority to devalue what the children themselves, in the meantime, have found most valuable. [1]
>
> The time! The time! Who's got the time? (March Hare in *Alice in Wonderland*, Lewis Carroll, 1865)

1. Introduction

Current standards and recommendations call for students to be much more active and self-directed in the doing of mathematics (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [2,3]). Students need autonomy and responsibility to think for themselves if they are to engage in mathematical practices such as constructing viable arguments and critiquing the reasoning of others or looking for

CONTACT Ian Whitacre iwhitacre@fsu.edu School of Teacher Education, Florida State University, 1114 W. Call St, Tallahassee, FL 32306-4459, USA

and expressing regularity in repeated reasoning [2]. This vision of mathematics learning, which positions students to explore mathematical ideas and make discoveries, allows space for play — the ultimate autonomous activity — and autonomy is key to intrinsic motivation [4]. People need the freedom to make choices, both at work and at school, in order to perform at their best when presented with novel tasks [4,5]. These are compelling reasons to allow for play in mathematics classrooms.

However, typical experiences of mathematics in U.S. public schools have long resembled work more than play [6]. The emphasis on standardized testing and accountability in the era of *No Child Left Behind* and now the *Every Student Succeeds Act* has heightened concerns that teachers are pressured to move quickly through curricula and teach to the tests rather than to provide opportunities for meaningful learning [7,8]. Students are rarely given substantial autonomy during in-class mathematical activities. Instead, both how they spend their time and the details of their mathematical work tend to be highly constrained [9]. Teachers are expected to engage students in innovative, standards-based activities, while being pressured in ways that constrain their abilities to do so [10]. Teachers may wish to teach in student-centred ways but feel like the March Hare in *Alice in Wonderland*: 'The time! The time! Who's got the time?'

Innovative high-tech software and hardware tools may help to relieve pressures on teachers by making it easier and more efficient to engage students in standards-based activities. In this study, we focus on the use of PhET interactive simulations (sims). PhET sims are designed for student accessibility, to encourage exploration and discovery, and lend themselves to student-centred instruction [11,12]. PhET sims provide students autonomy to explore mathematical concepts, test their understanding, and receive feedback within the context of a dynamic virtual environment [11].

In lessons involving PhET sims, there are advantages to allowing students to have a period of play with a sim before engaging them in more structured activities [13,14]. An initial play period offers students time to freely explore a sim, manipulate controls, ask questions, discover relationships, and generally become interested and actively engaged in the topic. This initial autonomous activity may contribute to students taking more ownership of their learning [13]. On the other hand, play by its nature is disruptive [15]. Free exploration with sims represents a sanctioned form of play within otherwise restrictive classroom environments. Thus, play may be at odds with teachers' priorities, including the institutional expectation that they must specify ahead of time the activities that will be completed during a lesson and keep up with a pacing guide. Therefore, there is an inherent 'struggle between order and chaos' [15, p.10] in lessons that include play.

The present study focuses on the tension between teaching within constraints, especially the need to keep pace, and allowing opportunities for divergent exploration of mathematical ideas. We investigated different versions of play facilitated by four mathematics teachers who used PhET sims. We examined distinguishing characteristics of the play phases of these lessons in light of the tensions inherent in the teachers' work.

2. Conceptual framework

The conceptual framework for this study is informed by perspectives concerning (a) play itself and (b) play in relation to teaching. Below we describe ideas from these areas and relate them to the phenomenon of sim play.

2.1. Sicart's theory of play

Sicart [15] conceptualizes play as contextual, carnivalesque, appropriative, disruptive, creative, personal, and autotelic. Play is contextual in that it has boundaries: there are players and non-players, there may be a designated field or other space for play, and there is a beginning and end to play. The broader context, including the culture, in which play takes place influences the nature of play. Cues signal possibilities for play, inviting play to begin or inviting non-players to join. In general, play involves rules, which may be either implicit or explicit.

Play is carnivalesque in that it may function subversively to mock or otherwise transform a situation; play has a sense of humour about it. Play is appropriative of spaces, linguistic forms, and cultural conventions. For example, a game of hide and seek may appropriate a library for a purpose determined by and known only to the players. Play can be disruptive. In some contexts, play is considered inappropriate, but it happens nonetheless. Play is creative. It often affords opportunities for imagination and improvization. Relatedly, play is personal. Players have room to express themselves in play. Play is autotelic, meaning that it is an end in itself [15].

Schools are meant to serve the purpose of educating students. So, play within classroom contexts ought to contribute to that broad purpose. The creative, personal, and autotelic nature of play make it conducive to motivation and learning; however, the carnivalesque, appropriative, and disruptive nature of play may threaten order in classroom contexts. Thus, there are inherent tensions in the role of play in education.

2.2. Play creates opportunities for mathematics learning

Young children are expected to play, so it seems natural to think about the role of play in early childhood mathematics learning, both in terms of allowing children to learn through play and capitalizing on opportunities for learning that arise during play. van Oers [16] describes how play creates opportunities for mathematical learning when children's actions are identified as mathematical by a teacher or more knowledgeable other. In the instructional approach that van Oers [16] describes, 'a teacher is always looking for meaningful teaching opportunities in the context of play that contributes to children's ability for participation and that opens new ways of thinking and learning' (p.30). While play may afford teaching opportunities in early childhood, it may be at odds with the norms of middle-school classrooms where school has become a place for work. To better understand the role of play in the middle-school settings, research must address how teachers can facilitate play in a context of competing priorities.

2.3. Play and teaching: competing priorities

In this study, we take a fundamentally pragmatic stance regarding teaching. We are concerned with what can be accomplished under all of the typical constraints associated with public school classrooms in the United States (e.g. pacing guides and the emphasis on test preparation). Given those constraints, interest in the pedagogical value of play leads to competing priorities. Play encourages students to explore, make discoveries, and generate their own ideas—and these are considered to be valuable opportunities that should be part of students' learning experiences [2]. Teachers, however, typically have predetermined curriculum maps and a constrained amount of time in which to get through a curriculum (e.g. [17]). In fact, since the passage of *No Child Left Behind*, administrators and school officials have raised their expectations for U.S. teachers to increase efficiency in the classroom and avoid frivolity [18]. Furthermore, teachers are concerned with maintaining a degree of control over their students' behaviour and activities [19,20]. Play offers a kind of freedom that may threaten to subvert or overthrow that control. So, although play has pedagogical merits in the interest of learning, it may also interfere with the business of teaching.

Bonawitz et al. [21] discuss the 'double-edged sword of pedagogy': On the one hand, pedagogy is intended to promote and guide learning; on the other hand, pedagogy can reduce intrinsic motivation and detract from learning. They found that when an adult demonstrated one function of a toy, children were less likely to discover other functions of the toy. Thus, when learners are instructed in how to use a tool that has the potential to be used for play, they may be less curious and less intrinsically motivated to explore, and they may make fewer discoveries as result.

3. Literature review

The above perspectives on play and its relationship to teaching and learning help us to conceptualize research concerning the role of sims in education. Below, we summarize literature on sims and play. We describe advantages of teaching with sims, and we describe the teaching practices and features of sims that contribute to students' learning.

3.1. The potential of sims to support learning

There are a wide variety of high-tech tools available to teachers, and these may be used for reasons that include making learning attractive to students and realizing education goals [22]. Here, we focus on *interactive simulations*: computer-based dynamic environments that model and allow users to interact with a concept, relationship, system, or phenomenon [23–25]. Sims, as defined here, have several common features: They are designed to be used interactively by students; make use of multiple, dynamic visual representations that support student connection making; focus on active knowledge construction; provide immediate feedback to users ([23–27]; and '[enhance, amplify, and guide] the cognitive process of learners' [28, p.142].

Sims can be advantageous in mathematics classrooms in a variety of ways. Students report enjoying working with these tools [23,24,27,28], and teachers report high levels of student engagement in lessons involving sims [28,29]. Sims support student learning of mathematics [23,27,29,30] and studies have shown significant gains in procedural skills, conceptual understanding, and problem solving [23,27,31,32]. Despite these potential advantages, it is crucial to consider how teachers facilitate lessons involving sims. Our interest in this study lies in how teachers facilitate play and the kinds of opportunities that students may be afforded, depending on teachers' choices.

3.2. Teaching with sims

While technology use can positively impact student learning of mathematics, this improvement is not automatic. There is a growing body of literature suggesting that what teachers do with computer technology in general, and with sims specifically, is key to promoting learning gains (e.g. [33–35]). In a meta-analysis of research on using technology for teaching mathematics, Li and Ma [36] found that the positive effect of technology was greater in classrooms when pedagogy was more aligned with constructivist principles (e.g. student-centred, problem-based) than with traditional instruction. Specific elements of instruction that appear to influence successful implementation of interactive sims include (a) sets of guiding questions for students to focus on as they work with the technology [11,28]; (b) peer discussion, either in small groups or as a whole class [37–39]; (c) opportunities for students to construct informal rules and develop their own understanding before rules and vocabulary are formally introduced [37–39]; (d) taking up and addressing incorrect answers using high-level moves such as revoicing and building on students' contributions [39]; and (e) time for students to consolidate ideas and understanding through prompted reflection, culminating discussions, and/or writing [28,37,40].

3.3. PhET sims

The PhET Interactive Simulations Project (https://phet.colorado.edu) develops and studies the use of sims for science and mathematics education. These freely available sims are designed to be flexible tools for teachers: they can be used in a variety of ways (e.g. during labs, lecture, or group work) and to teach a wide range of topics.

The design features of PhET sims lends themselves to play. Perhaps most notable of these features is *implicit scaffolding*, a type of cuing and guidance that includes intentional decisions by designers about what actions the environment affords, what actions it constrains, and what kinds of feedback it gives in response to users' actions [41,42]. This is done mainly without the use of text or explicit instructions, and is intended to influence students' interactions with the sim and to encourage them to engage in productive ways in order to support students' progression in understanding concepts [14,43]. At the same time, it is done subtly, so that students are guided without feeling guided [44].

For example, in *Function Builder* (Figure 1), visual cues support students to discover that they can drag inputs into the function machine and observe the corresponding outputs. Controls such as the ability to toggle between one rule and multiple rules are apparent through the use of icons, highlighting, and color-coding. Even the layout of the controls is implicitly scaffolded so that students interact with controls in desired ways: students tend to move from left to right as they explore, so designers placed items they want students to interact with first on the left, and feedback is displayed on the right. Thus, PhET sims enable students and teachers to freely explore and use PhET sims with minimal to no directions [45].

3.4. Sims and play

Sims are not games, although they may be game-like or even include game screens. Neither does play require games. As Sicart [15] says, 'Games don't matter' (p.2). It is play itself that matters, and play can take a variety of forms and occur in virtually any context. PhET sims, as explained above, afford and invite play. Rather than instructing students in exactly what to do, PhET sims are designed to stimulate interest by providing subtle cues and then feedback in response to the user's curious clicks.

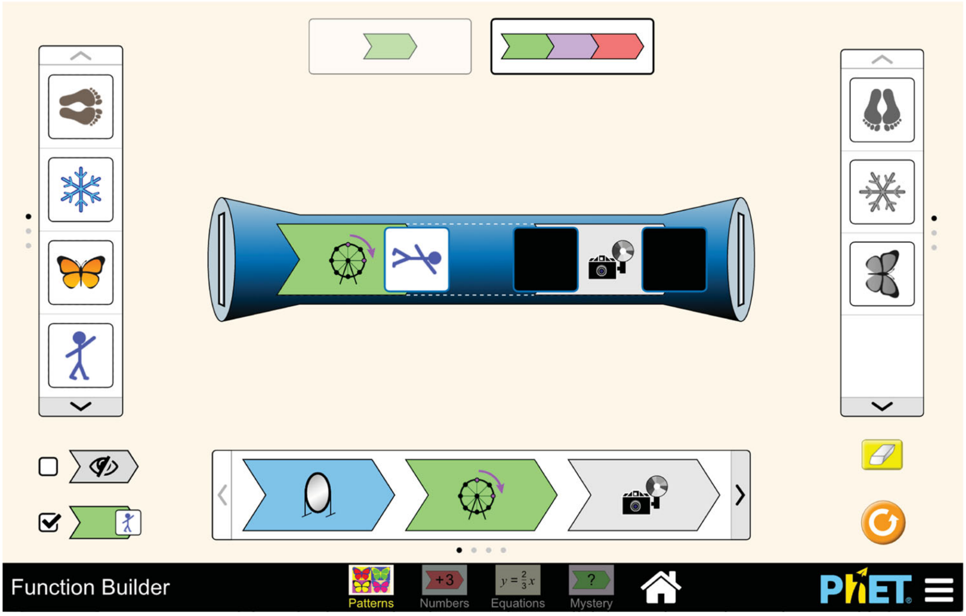

Figure 1. Function builder: patterns.

There seem to be advantages to including an initial play period in sim lessons. Podolefsky et al. [13] found that a science lesson that began with play featured significantly more focus on student ideas and content than the same lesson taught without play. Play, however, does not automatically support learning [46]. As with any instructional activity, teachers have choices to make regarding how the activity will be introduced and facilitated.

How might play be effectively incorporated into mathematics instruction involving PhET sims? The literature does not make clear how teachers can balance competing priorities by allowing for play in sim-based lessons, while steering lessons in predetermined directions and keeping up with prescribed pacing.

4. Methods

To address the above gap in the literature, we investigated how mathematics teachers facilitated play in their sim-based lessons and for what purposes. In particular, we asked the following research questions:

1. What are distinguishing characteristics of the play phases of middle-school mathematics lessons involving PhET sims?
2. What combinations of characteristics do we observe in the play phases of sim-based lessons?
3. What distinct profiles of play do we find, and what purposes does play serve for each profile?

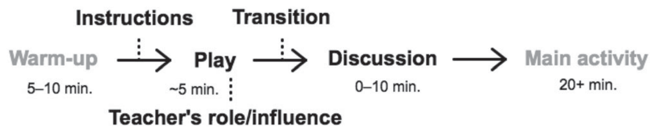

Figure 2. Anatomy of a sim-based lesson: before, during, and after play.

4.1. Setting

Data collection took place at a middle school in the Southeastern United States. The school was a public charter school associated with a university, and its demographics were representative of the state (52% White, 30% Black, and 10% Latina/o students, with 21% of students eligible for free or reduced-price lunch). Four middle-school mathematics teachers and their students participated in the study. We refer to the four teachers by the pseudonyms Angelica, Dorothy, Elizabeth, and Penelope. Each teacher had at least seven years of teaching experience, but they were new to teaching with PhET sims. Participating classes were selected based on scheduling considerations and the intention of observing a variety of mathematics lessons. They included Grade 6 Intensive, Grade 7, Grade 7 Advanced, Grade 8 Pre-algebra, and Grade 9 Algebra 1.

The teachers were introduced to PhET sims in two workshops, one led by the first author and one led by a PhET curriculum specialist. The teachers were given sample activity sheets from a sim-based science lesson and a sim-based mathematics lesson, as well as recommendations and documents that provided guidelines for the design of sim-based activities (http://phet.colorado.edu/en/teaching-resources/activities-design). These recommendations were based on research conducted previously in mathematics and science and included taking advantage of sim features, building in time for play, and structuring activity sheets to scaffold students' investigations during the main lesson activity and classrooms (e.g. Authors [24]). Although the research team provided guidance and feedback, the teachers planned their own lessons and made independent choices as to when and how they would use sims in their instruction.

4.2. Data collection and analysis

During the 2015–2016 school year, we observed and video recorded 15 sim-based mathematics lessons[1] (2–5 lessons per teacher).

We describe our methods of analysis related to each research question.

4.2.1. Analysis for Research Question 1

To identify distinguishing characteristics of the play phases of lessons, we watched each of the 15 lesson videos and identified segments that were considered to have taken place before, during, or after play. Figure 2 represents the anatomy of a sim-based lesson, focusing on these segments of activity. The lessons typically included a warm-up activity prior to play. Some included a discussion after play, whereas others proceeded directly to the main activity.

We began by open coding [47] each of these periods of activity with an interest in the tension inherent in mathematics lessons involving play. Initial analysis led to the identification of four dichotomous characteristics: (1) Instructions for play—open or lightly constrained; (2) Teacher's role or influence during play—unguided or guided; (3) Products of play (e.g. students' observations, discoveries, or questions)—visible or invisible; and (4) Discussion of products of play (if applicable)—focused or meandering. (Details of each characteristic are presented in the Results section.) All of the videos were coded for these characteristics.

4.2.2. Analysis for Research Question 2

The results of the above coding of characteristics were used to identify combinations of characteristics that were observed in our data set (e.g. Open-Unguided-Visible-Meandering). These combinations were further compared and contrasted to define profiles of play.

The first author coded all of the videos. The second author coded 9 of the 15 videos (60% of the data). We measured inter-rater reliability by comparing the 32 coding decisions that both raters were required to make for those 9 videos. Raters initially agreed on 28 of 32 decisions, for 87.5% inter-rater reliability. The disagreements were discussed and resolved, resulting in 100% agreement after discussion. Levels of inter-rater agreement above 80% are commonly regarded as very high or excellent [48].

4.2.3. Analysis for Research Question 3

The above results informed our analysis of profiles of play. We present three profiles, which can clearly be ordered along a continuum of priorities from keeping pace to foregrounding students' thinking. For each profile, we analysed apparent purpose(s) of play, taking into account each of the segments and characteristics of play in the lessons that fit that profile. A process of constant comparative analysis led to three codes for purposes of play: (a) Become familiar with the sim, (b) Make and share discoveries, and (c) Make and share *relevant* discoveries. Each session of play was coded for these purposes. The purposes were then used to characterize the three profiles of play.

5. Results

We present results in order of the research questions.

5.1. Characteristics of sim play

In answer to Research Question 1, we identified four dichotomous characteristics of play. Below, we provide examples to illustrate each characteristic.

5.1.1. Instructions for play

The instructions for sim play were either open or lightly constrained.[2] *Open instructions* sounded like, 'Just play with the sim' or 'Click on things, drag them, see what happens.' These instructions constituted open invitations to experiment and become familiar with the sim. *Lightly constrained instructions*, by contrast, place some restriction on play, such as 'We're going to put the purple point at the origin and leave it there for the rest of the lesson.' Note that lightly constrained instructions still leave room for sim play; they stop far short of directing students' specific actions. Heavily constrained instructions would disqualify the activity from being considered play at all.

5.1.2. Teacher's role or influence during play

During play, students' activity may be unguided or guided by the teacher. This distinction looks past the initial instructions and focuses on the teacher's actions during play and how these actions influence students' play with the sim. In *unguided play*, the teacher typically circulates the room, focusing on classroom management and keeping students on task. She may influence students in the sense of encouraging them to engage with the sim, but the teacher has little influence over how students engage with the sim.

In *guided play*, the teacher engages in deliberate and substantive interactions with students during play. For example, the teacher may ask particular students (not necessarily all students) what they noticed about the sim and then follow up by pressing students to be more precise in their descriptions or to ask why they think the sim works in those ways. Note that students are still playing with the sim. For the most part and during the vast majority of the play period, they are free to explore the sim in divergent ways; however, the teacher–student interactions may nudge students to focus on particular aspects of the sim and to progress in their thinking about those aspects.

5.1.3. Products of play

While students were typically asked to record their discoveries, observations, or questions during the play period, these products of play were not always made apparent, or visible, to the whole class. Thus, we describe the products of play as *visible* or *invisible*. When products of play are made *visible*, students' activity and ideas explored during play are solicited and used explicitly in a subsequent activity. For example, the teacher may call on students to share their discoveries, making those a visible product of the play process. By contrast, in lessons with *invisible* products of play, students' discoveries or questions generated during play remain private. The teacher may or may not have gathered some information about students' ideas from wandering the classroom, but there is no teacher facilitation component that makes these discoveries public to the class. Students may be instructed to begin a worksheet or move on to the next task without any opportunity to share and make visible ideas that came up for them during play, and play seems disconnected from students' subsequent activities.

By contrast, when the products of play were invisible, play ended without opportunities for students to contribute their discoveries or ask questions. Even if students generated questions or discoveries during play, these did not serve any shared purpose for the class. Students were asked to jot down a few points on their worksheet and then moved on.

5.1.4. Discussion of products of play

If there is a discussion that follows sim play and relates to products of play, such discussion may be categorized as *focused* or *meandering*. In play sessions with visible products, students typically share discoveries in whole-class discussion. Such discussion may provide valuable opportunities for students to contribute ideas, questions, and arguments. At the same time, these contributions may or may not be relevant to the learning goals for the lesson. In a *meandering* discussion, students' contributions are free to diverge. These may relate to the sim and/or to mathematics but not necessarily to the day's learning goals, and thus may not advance the lesson. When discussion is *focused*, there is some structure to the discussion so that students' contributions facilitate progress towards the learning goals. For example, the teacher may guide the discussion with a set of questions or call on particular students with whom she interacted during play. These questions may still be open, so that there remain ample opportunities for students to contribute a variety of ideas, but students' contributions tend to converge on topics and questions that are relevant to the day's lesson.

Meandering discussions, by contrast, followed students' ideas but did not steer the discussion in the direction of specific learning goals. The discussion was guided by whatever students happened to say, whether or not it related to the lesson's topic.

5.2. Profiles and purposes of play

Research Questions 2 and 3 investigated the types and purposes of profiles of the play phase. Given the dichotomous characteristics that we identified (Open/Lightly Constrained, Guided/Unguided, Visible/Invisible, and Focused/Meandering—which apply only to those lessons in which there is a play-based discussion), there are 12 possible combinations of these characteristics. Not all of these combinations were observed in our data set. We report here only the combinations that we observed, and we characterize three distinct profiles. Table 1 summarizes the combinations of characteristics of the play phases of lessons in our data set, which range on a continuum from keeping pace to foregrounding students' ideas.

We focus here on three profiles, which were clearly distinguishable in our data. We define the profiles based only on those characteristics that appeared to have practical implications. In particular, lightly constrained versus open instructions did not appear to affect play in our data set, so we do not regard characteristics of instructions as a defining feature of the profiles. (We further address these implications in the *Discussion* section.).

5.2.1. A timed sprint

One profile that we observed had the characteristics Unguided-Invisible. Here, play is unguided (e.g. the teacher circulates the room, making sure students are on task); and the products of play are invisible (e.g. the activity ends abruptly with the transition to a worksheet); accordingly, there is no discussion related to the play that took place. This profile

Table 1. Characteristics of the play phases of the sim-based lessons in our data set.

Lesson	Instructions for play	Teacher role during play	Products of play	Discussion of products of play
Angelica Lesson 1a	Lightly constrained	Unguided	Visible	Meandering
Angelica Lesson 1b	Lightly constrained	Unguided	Visible	Meandering
Dorothy Lesson 1a	Open	Unguided	Invisible	N/A
Dorothy Lesson 1b	Open	Unguided	Invisible	N/A
Dorothy Lesson 2a	Open	Unguided	Visible	Meandering
Dorothy Lesson 2b	Open	Unguided	Visible	Meandering
Dorothy Lesson 3	Open	Unguided	Visible	Meandering
Elizabeth Lesson 1	Open	Guided	Visible	Focused
Elizabeth Lesson 2	Open	Guided	Visible	Focused
Elizabeth Lesson 3	Lightly constrained	Guided	Visible	Focused
Elizabeth Lesson 4	Open	Guided	Visible	Focused
Penelope Lesson 1a	Open	Unguided	Invisible	N/A
Penelope Lesson 1b	Open	Unguided	Invisible	N/A
Penelope Lesson 2a	Open	Unguided	Invisible	N/A
Penelope Lesson 2b	Open	Unguided	Invisible	N/A

Letters a and b refer to multiple enactments of the same lesson plan, whereas distinct numbers refer to different lesson plans.

emphasizes moving through the lesson efficiently and using play only to familiarize students with the sim. For this reason, we describe it as *A Timed Sprint*. This profile leans heavily in the direction of keeping pace by carrying out the teacher's plan. Like a timed sprint, the emphasis is on reaching a predetermined destination as quickly as possible, thus offering little autonomy or opportunities for exploration.

5.2.2. Wandering exploration

A second profile that we observed had the characteristics Unguided-Visible-Meandering. In this profile, play was monitored but not guided; there were visible products of play in the form of students' discoveries, and subsequent discussion focused on students' discoveries, whether or not these were relevant to the lesson plan. We characterize this profile as *Wandering Exploration*. It highly prioritizes opportunities for students to make discoveries and to share and discuss these. This exploration occurs with little apparent concern for keeping pace or completing the planned activity. In other words, the apparent purpose is to explore and discover something, rather than to reach a predetermined destination. The two profiles above represent opposite extremes, each decidedly favoring one of the two competing priorities (see Figure 3).

5.2.3. Hiking with a guide

The third profile achieves a balance between the competing priorities. This profile has the characteristics Guided-Visible-Focused. In this route through the play phase, play is guided (e.g. teacher interacts with individual students or small groups of students about specific details of the sim); the products of play are visible (e.g. play is followed by a discussion of students' discoveries made during play); and the discussion is focused (e.g. the teacher asks specific questions or steers the discussion towards relevant mathematical ideas). Examples of this profile provided ample opportunities for students to make discoveries and contribute ideas, and yet, such lessons proceeded efficiently, and the teacher steered activity in the direction of the learning goals. Given its balanced nature, we characterize this profile as *Hiking with a Guide*. Hiking provides plenty of opportunity for making and sharing discoveries and learning new things. Simultaneously, a guide helps keep the hike on course and increases the probability of reaching a predetermined destination.

In each of the profiles, play gives students the opportunity to become familiar with the sim. Students experiment with controls and find out their functions. They may attend to features of the design of the sim and/or aspects of the mathematics concepts involved. In the profile that we call *A Timed Sprint*, play only serves the purpose of familiarizing students with the sim. Play happens relatively quickly, and then the lesson moves forward. In all versions of sim play, students may make discoveries. However, discoveries that do not become visible products of play serve no apparent purpose. In the profile *Wandering*

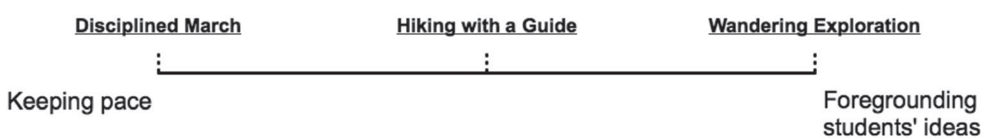

Figure 3. Profiles of the play phase in relation to the two competing priorities.

Exploration, play serves the additional purpose that students make and share discoveries. Alternatively, as noted above, these discoveries may or may not be relevant to the goals of the lesson. In *Hiking with a Guide*, by contrast, students make and share relevant discoveries. The relevance of these discoveries is influenced by the teacher's subtle guidance in interactions with students during play, as well as by choices that the teacher makes when facilitating the discussion following play.

6. Discussion

We extend our findings concerning characteristics of the play phases of sim-based lessons, combinations of these characteristics, and profiles and purposes of play to begin to develop a theory that addresses how mathematics teachers can productively incorporate sim play into their lessons. In reflecting on our data, we have come to view teachers' choices in planning and enacting the play phases of sim-based lessons as managing the tension between divergent and convergent modes of activity or what Sicart [15] describes as 'struggle between order and chaos' (p.10). During play, the focus of students' activity is allowed to diverge to some degree as each student explores different sim features. In contrast, by the time of the main task of the lesson, student activity tends to converge. By providing more specific instructions for the activity (often in the form of a worksheet with numbered tasks), teachers attempt to focus students on particular sim features and related mathematical ideas. We use these notions to frame teacher's choices regarding play.

In giving instructions for play, teachers begin to set the course for students' activity with the sim in their choice to provide relatively light or heavy constraints. Light constraints function like boundaries for play, much like the edges of a playground. There is still plenty of room to play within those boundaries, because students have autonomy and many options available within that space.

Figure 4 depicts a contrast between more and less divergent versions of play. The dotted segments represent the period before play, including the initial instructions for play. The solid segments represent activity during play, with the spread of the angle meant to convey the extent of divergence in students' activities with the sim. Our concern here is with the intentions or likely consequences of teachers' choices. Open instructions welcome highly divergent play, whereas constraints on play may offer less opportunity for divergence in students' sim activities. In all cases, there are some boundaries on appropriate use. For example, when a teacher directs students to use a specific PhET sim, the students are expected to play with that sim and not with other sims at the PhET website.

As play takes place, teachers may shape the nature of play through their interactions with students. In the metaphor of playing on a playground, teachers supervise the activity

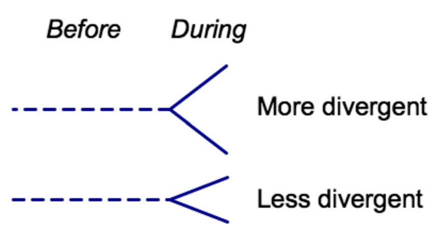

Figure 4. Varying degrees of divergence in play.

that takes place. Teachers may actively guide play, as a coach might do, or they may simply monitor the activity from the sidelines. Guidance during play, like instructions, may narrow the divergence of students' play, which is not necessarily undesirable unless taken too far. As noted previously, students following step-by-step instructions for what to do with a sim would not be playing at all. Like Elizabeth, teachers using play to prepare students for a focused discussion may wish to lightly rein in divergence during play so that students attend to certain features of the sim or ideas related to these.

Sim-based lessons may contrast in terms of the activity that follows play. Teachers may decide to make students' discoveries or questions public in a whole-class discussion, thus connecting that discussion to the play that preceded it and accomplishing convergence gradually. Alternatively, teachers may interrupt play and move to a worksheet activity that is disconnected from play and requires immediate convergence.

Finally, teachers may welcome all student contributions to whole-class discussion and entertain these without concern for the focus of the lesson, or they may narrow those discussions to converge on sim features and mathematical ideas that are especially relevant to the day's lesson. Ultimately, given the realities of standards, curricula, pacing guides, and test preparation that are typical of U.S. schools, it seems impractical for teachers to spend valuable time on meandering discussions. There is a need to leverage students' ideas, questions, and discoveries in productive directions, so that the priority of keeping pace is not sacrificed. Figure 5 depicts the contrasts between interrupted play, which is characteristic of lessons with invisible products, and lessons in which the class transitions smoothly from play to a discussion of the products of play. The figure further depicts the contrast between focused and meandering discussions. Whereas focused discussions involve relatively efficient convergence to a focal topic, meandering discussions lack this focus and may extend for too long.

Throughout the play phase of sim-based lessons, teachers face the challenge of managing divergent and convergent modes of activity. Neither extreme seems optimal. Rapid or abrupt convergence prioritizes keeping pace at the expense of foregrounding students' ideas, whereas prolonged divergence prioritizes foregrounding students' ideas at the expense of keeping pace. Thus, successfully navigating play involves a balancing act. The profile of Hiking with a Guide characterizes the role of a teacher who manages this process effectively.

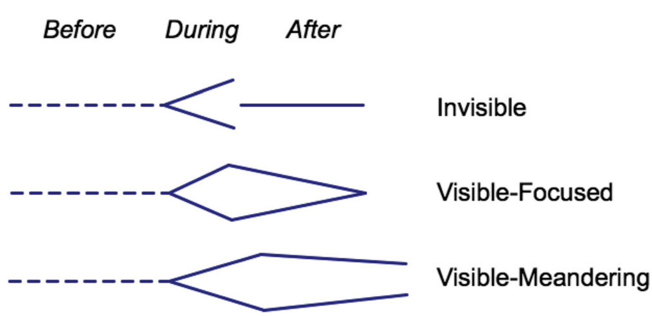

Figure 5. Varying approaches to convergence following play.

7. Conclusions

Below, we briefly summarize our findings. We then highlight this article's contributions to the literature. We end with implications.

7.1. Summary of findings

This study represents an investigation into the challenging but important question of how to incorporate play into mathematics pedagogy. On the one hand, play offers autonomy and creates opportunities to foreground students' thinking. On the other hand, play is potentially disruptive and threatening to a teacher's ability to keep pace. In the context of play with PhET sims in middle-school mathematics lessons, we found that both lightly constrained play and open play with PhET sims appeared to equally afford divergent exploration of the sim. Guided versus unguided play appeared to be more focused and relevant to the topic of the lesson, with no apparent disadvantage. Visible versus invisible products of play afforded greater opportunities for student contributions. Additionally, focused versus meandering discussions of the products of play contributed to more coherent and efficient lessons.

We highlighted three profiles of play, based on combinations of the above characteristics, and we ordered these along a continuum of priorities, from keeping pace to foregrounding students' ideas. We identified one particular profile of play, termed *Hiking with a Guide*, which best managed to balance those competing priorities.

7.2. Contributions to the literature

These findings advance what is known about how mathematics teachers may incorporate sim play into their instruction. We identified different ways in which teachers may approach the play phase of a lesson, and we examined the practical implications of these. While the culture of public schools in the United States and elsewhere puts pressure on teachers to keep pace with curriculum maps [18], research-based recommendations and policy documents emphasize recognize the importance of providing opportunities for students to explore mathematics and generate ideas [2,3], as well as the importance of affective aspects of students' mathematical experiences [9]. Despite these recommendations, the literature has offered little help in answer to the question of how teachers can successfully balance competing priorities.

We identified the purposes that play served as a way of distinguishing the three profiles of the play phase that our analysis revealed. Hiking with a Guide enabled students to become familiar with the sim and to make and share relevant discoveries. Thus, despite concerns over the compatibility of play with teaching, we identified a way in which teachers can incorporate play with interactive computer simulations into middle-school mathematics lessons that strikes a balance between the competing priorities of keeping pace and foregrounding students' ideas.

In lessons involving sims, there are advantages to allowing students the opportunity to play. Providing students to become familiar with the controls in a sim and their functions through play can support more substantive interactions later in the lesson [13]. In the context of mathematics lessons involving PhET sims, we found that play could serve the additional purpose of affording students the opportunity to make and share discoveries, thus contributing to the goal of foregrounding students' ideas.

7.3. Implications

The choices that teachers make when incorporating play into sim-based lessons pull in the direction of keeping pace or in the direction of foregrounding students' ideas. Our findings inform recommendations regarding how teachers can balance these competing priorities. By providing subtle guidance during play, making students' discoveries visible, and facilitating discussions focused on relevant discoveries, teachers can balance the competing priorities and make the most of sim play. These findings contribute to resources that provide practical recommendations for teachers who are interested in using interactive sims, or similar open exploratory tools. Future research might shift focus from teachers' facilitation of play to students' play experiences, given the variations that we observed, and examine the influence of these experiences on students' attitudes towards mathematics and on their learning of the relevant content.

Notes

1. When the same sim was used in related ways for two consecutive days, this was considered one lesson. Enactments of the same lesson plan with two different groups of students were each counted as a lesson, due to the opportunities for variability between enactments.
2. Instructions for play often also included students writing down observations made during play, but this aspect is orthogonal to the distinction between open and lightly constrained.

Acknowledgments

We thank the teachers for their participation in this study. We also thank Jeremy Roschelle, Kathy Perkins, Emily B. Moore, Lama Jaber, and Miray Tekkumru-Kisa for their helpful feedback on earlier drafts of this manuscript. Any opinions, findings, conclusions, and recommendations expressed in this material are those of the authors and do not necessarily reflect the views of NSF.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This material is based upon work supported by the National Science Foundation (Division of Research on Learning in Formal and Informal Settings) [1503510].

References

[1] Hawkins D. Messing about in science. Sci Child. 1965;2(5):5–9.
[2] National Governors Association Center for Best Practices & Council of Chief State School Officers. Common core state standards for mathematics. Washington (DC): National Governors Association Center for Best Practices & Council of Chief State School Officers. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.
[3] National Council of Teachers of Mathematics. Principles to actions: ensuring mathematical success for all. Reston, VA: National Council of Teachers of Mathematics; 2014.

[4] Gagné M, Deci EL. Self-determination theory and work motivation. J Organ Behav. 2005;26(4):331–362.
[5] Deci EL, Vallerand RJ, Pelletier LG, et al. Motivation and education: the self-determination perspective. Educ Psychol. 1991;26(3–4):325–346.
[6] Stigler JW, Hiebert J. The teaching gap: best ideas from the world's teachers for improving education in the classroom. New York: Free Press; 1999.
[7] Posner D. What's wrong with teaching to the test? Phi Delta Kappan. 2004;85(10):749–751.
[8] Roach R. Teaching to the test. Diverse: Issues High Educ. 31(3):32–36.
[9] National Research Council. Adding it up: helping children learn mathematics. Kilpatrick J, Swafford J, Findell B, editors. Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press; 2001.
[10] Nadelson LS, Briggs P, Bubak K, et al. The tension between teacher accountability and flexibility: the paradox of standards-based reform. Teach Educ Pract. 2012;25(2):196–220.
[11] Hensberry KKR, Paul AJ, Moore EB, et al. PhET Interactive simulations: new tools to achieve common core mathematics standards. Hershey (PA): IGI Global; 2013. (Polly D, editor. Common core mathematics standards and implementing digital technologies).
[12] Perkins KK, Loeblein PJ, Dessau KL. Sims for science: powerful tools to support inquiry-based teaching. Sci Teach. 2010;77(7):46–51.
[13] Podolefsky NS, Rehn D, Perkins KK. Affordances of play for student agency and student-centered pedagogy. American institute of physics conference series, Vol. 1513, pp. 306–309; 2013.
[14] Moore EB, Herzog TA, Perkins KK. Interactive simulations as implicit support for guided-inquiry. Chem Educ Res Pract. 2013;14(3):257–268.
[15] Sicart M. Play matters. Cambridge: MIT Press; 2014.
[16] van Oers B. Emergent mathematical thinking in the context of play. Educ Stud Math. 2010;74:23–37.
[17] Vannest KJ, Hagan-Burke S. Teacher time use in special education. Remedial Spec Educ. 2009;31:126–142.
[18] Metzker B. Time and learning. ERIC Digest. Available from: http://files.eric.ed.gov/fulltext/ED474260.pdf; 2003.
[19] Brunetti GJ. Why do they teach? A study of job satisfaction among long-term high school teachers. Teach Educ Quart. 2001;28(3):49–74.
[20] Pearson LC, Moomaw W. The relationship between teacher autonomy and stress, work satisfaction, empowerment, and professionalism. Educ Res Quart. 2005;29(1):37–53.
[21] Bonawitz E, Shafto P, Gweon H, et al. The double-edged sword of pedagogy: instruction limits spontaneous exploration and discovery. Cognition. 2011;120:322–330.
[22] Heitink M, Voogt J, Verplanken L, et al. Teachers' professional reasoning about their pedagogical use of technology. Comput Educ. 2016;101:70–83.
[23] Hensberry KKR, Moore EB, Perkins KK. Effective student learning of fractions with an interactive simulation. J Comput Math Sci Teach. 2015a;34(3):273–298.
[24] Hensberry KKR, Moore EB, Perkins KK. Using technology effectively to teach about fractions. APMC. 2015b;20(4):19–25.
[25] D'Angelo C, Rutstein D, Harris C, et al. Simulations for STEM learning: systematic review and meta-analysis. Menlo Park, CA: SRI International; 2014.
[26] Moyer PS, Boylard JJ, Spikell MA. What are virtual manipulatives? Teach Child Math. 2002;8(6):372–377.
[27] Reimer K, Moyer PS. Third graders learn about fractions using virtual manipulatives: a classroom study. J Comput Math Sci Teach. 2005;24(1):5–25.
[28] Kay R. Exploring the impact of web-based learning tools in middle school mathematics and science classrooms. J Comput Math Sci Teach. 2011;30(2):141–162.

[29] Kay RH, Knaack L. Exploring the use and effect of learning objects in middle school classrooms. Can J Learn Technol. 2008;34(1):51–73.
[30] Liu M, Bera S. An analysis of cognitive tool use patterns in a hypermedia learning environment. Educ Technol Res Dev. 2005;53(1):5–21.
[31] Bottge BA, Grant TS, Stephens AC, et al. Advancing the math skills of middle school students in technology education classrooms. NASSP Bull. 2010;94(2):81–106.
[32] Roschelle J, Shechtman N, Tatar D, et al. Integration of technology, curriculum, and professional development for advancing middle school mathematics: three large-scale studies. Am Educ Res J. 2010;47(4):833–878.
[33] Dalton SK, Hegedus S. Small group to whole group sharing of ideas in an elementary geometry class utilizing technology. Martinez M, Superfine A, editors. Proceedings of the 35th Annual Meeting of the North American chapter of the international group for the psychology of mathematics education; Chicago, IL: University of Illinois at Chicago; 2013. pp. 1133–1140.
[34] Drijvers P. Digital technology in mathematics education: why it works (or doesn't). Proceedings from The 12th International Congress on Mathematical Education (ICME). Seoul, South Korea; 2012.
[35] Gresalfi MS. Technology in mathematics education: a discussion of affordances. Martinez M, Superfine A, editors. Proceedings of the 35th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education; Chicago, IL: University of Illinois at Chicago; 2016. pp. 16–28.
[36] Li Q, Ma X. A meta-analysis of the effects of computer technology on school students' mathematics learning. Educ Psychol Rev. 2010;22(3):215–243.
[37] Suh J, Moyer PS, Heo HJ. Examining technology uses in the classroom: developing fraction sense using virtual manipulative concept tutorials. J Interact Online Learn. 2005;3(4): 1–22.
[38] Vahey P, Lara-Meloy T, Moschkovich J, et al. Representational technology for learning mathematics: an investigation of teaching practices in Latino/a classrooms. Presented at the Learning in the Disciplines: Proceedings of the 9th International Conference of the Learning Sciences, Chicago, IL; 2010.
[39] Zahner W, Velazquez G, Moschkovich J, et al. Mathematics teaching practices with technology that support conceptual understanding for latino/a students. The Journal of Mathematical Behavior. 2012;31(4):431–446.
[40] Vahey P, Werner A, Roy G, et al. Consolidation activities in technology-based environments: different teacher approaches. In: Martinez M, Superfine A, editors. Proceedings of the 35th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Chicago, IL: University of Illinois at Chicago; 2013. p. 1085–1088.
[41] Podolefsky NS, Moore EB, Perkins KK. Implicit scaffolding in interactive simulations: design strategies to support multiple educational goals. arXiv 2013. 1306.6544 [Physics]. Retrieved from http://arxiv.org/abs/1306.6544
[42] Podolefsky NS, Perkins KK, Adams W. Factors promoting engaged exploration with computer simulations. Phys Rev Spl Top Phys Educ Res. 2010;6(2):20117.
[43] Ares N, Stroup WM, Schademan AR. The power of mediating artifacts in group-level development of mathematical discourses. Cogn Instr. 2009;27(1):1–24.
[44] Paul A, Podolefsky N, Perkins K. Guiding without feeling guided: implicit scaffolding through interactive simulation design. Proceedings of the 2012 physics education research conference; 2012.
[45] Adams WK, Reid S, LeMaster R, et al. A study of educational simulations part I – engagement and learning. J Interact Learn Res. 2008;19(3):397–419.
[46] Rieber LP, Noah D. Games, simulations, and visual metaphors in education: antagonism between enjoyment and learning. EMI Educ Media Int. 2008;45(2):77–92.
[47] Corbin A, Strauss J. Basics of qualitative research: techniques and procedures for developing grounded theory. 3rd ed. Thousand Oaks, CA: Sage Publications; 2008.
[48] Gwet KL. Handbook of inter-rater reliability: the definitive guide to measuring the extent of agreement among raters. 4th ed. Gaithersburg, MD: Advanced Analytics; 2014.