Examining the Effects of Math Teachers' Circles on Aspects of Teachers' Mathematical Knowledge for Teaching

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Abstract

Math Teachers' Circles have been spreading since their emergence in 2006. These professional development programs, aimed primarily at middle-level mathematics teachers (grades 5-9), focus on developing teachers' mathematical problem solving skills, in line with the Common Core State Standards – Standards of Mathematical Practice. Yet, to date, anecdotal evidence and self-report data has been the primary measure of their effectiveness. This study examines the results of a three-site administration of the Learning Math for Teaching instrument, a multiple-choice instrument designed to measure aspects of Mathematical Knowledge for Teaching. Results indicate that Math Teachers' Circles are impacting teachers' performance on the Number Concept and Operation subsection, leading to implications for future research.

Keywords: mathematical problem solving, mathematical knowledge for teaching, Math Teachers' Circles, Common Core State Standards, Standards of Mathematical Practice, teacher professional development.

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Problem solving and critical thinking are cited globally as among the most important skills for college readiness, education at the college level, and participation in the 21st century workforce (Association of American Colleges & Universities, 2007; The Conference Board et al., 2006; Conley, 2007; National Governors Association, Council of Chief State School Officers, & Achieve, Inc., 2009; Partnership for 21st Century Skills, 2008; Robinson, Garton, & Vaughn, 2007; Yang, Webster, & Prosser, 2011). Indeed, the ability to make sense of problems and persevere in solving them is the first of the mathematical practice standards included in the Common Core State Standards (CCSS; Common Core State Standards Initiative, 2010), which have now been adopted by 45 of the 50 states in the United States.

As in-service teachers throughout the United States work to implement the new requirements of the CCSS, research states that effective professional development needs to be considered to support teachers (Schifter & Granofsky, 2012). While there is not universal agreement about what should be included in professional development (Borko, 2004; Guskey, 2003; National Mathematics Advisory Panel, 2008; Wayne, Yoon, Zhu, Cronen, & Garet, 2008), the consensus suggests that it should include opportunities for teachers to deepen their subject-matter knowledge (Cohen & Hill, 2002; Garet, Porter, Desimone, Birman, & Yoon, 2001; Kennedy, 1998; Porter, Garet, Desimone, & Birman, 2003). In terms of the CCSS, effective professional development could also support teachers by helping them implement the standards related to the practices of mathematics, including mathematical problem solving.

The Math Teachers' Circle (MTC) model is a relatively new (since 2006) form of professional development that emphasizes developing teachers' understanding of and ability to engage in the practice of mathematics, particularly mathematical problem solving, in the context of significant mathematical content. The core activity of MTCs is regular meetings focused on mathematical exploration, led by mathematicians or co-led by mathematicians and teachers.

Because they are typically sustained over multiple years, MTCs become communities in which both teachers and mathematicians participate and share their expertise (cf. Ball, 1999; Beckmann, 2011a, b; Lave & Wenger, 1998). The MTC model is also an example of a professional development program that has been scaled up to involve multiple facilitators at multiple sites. There are currently approximately 53 active MTCs in 31 states. Only two of these groups are led by originators of the MTC model, and a majority have met for multiple years, providing evidence of sustainability of the model. Thus, they have the potential to contribute to our knowledge of how professional development can be scaled up effectively (Borko, 2004).

Although MTCs are growing in popularity, little is known about their impact on teachers beyond anecdotal reports. This paper reports the results of the first formal study of MTCs. To our knowledge, it is also one of only two studies (along with Bell et al., 2010) to investigate the effects on mathematical knowledge for teaching of a single professional development program aimed at classroom teachers that has been implemented at multiple sites with multiple facilitators. We focused on three U.S. sites that are implementing MTCs for middle-level mathematics teachers (grades 5-9). Our research question was whether participation in a MTC affects teachers' mathematical knowledge for teaching. We hypothesized that by practicing mathematics themselves—participating actively, articulating mathematical ideas, critiquing the reasoning of others, and generally developing their mathematical habits of mind—teachers would increase their mathematical knowledge for teaching.

The Math Teachers' Circle Model

The Math Teachers' Circle (MTC) model was developed in 2006 at the American Institute of Mathematics (AIM), one of the eight National Science Foundation (NSF) mathematical sciences research institutes in the United States. The model is loosely based on the Eastern European model of Math Circles for K-12 students that has grown in popularity in the U.S. in recent years (cf. Shubin, 2006). The logistics of how the model is implemented vary depending on local circumstances, but all MTCs focus on problem solving in the context of significant mathematical content, draw on the content expertise of mathematicians, and have high levels of interactivity among participants and session leaders. The problems that form the basis of MTC activities generally have multiple entry points and are accessible to a wide range of teachers. Teachers spend considerable time working with each other on the problems. The primary focus is on the teachers as active mathematical learners who are developing their own problem-solving and mathematical reasoning skills.

The Math Teachers' Circle model was developed for middle-level mathematics teachers. Mounting evidence from national and international assessments indicates that few U.S. students are proficient in creative problem solving and mathematical reasoning activities, with deficiencies becoming significant in middle school (Gonzales et al., 2009; National Center for Education Statistics, 2009; Organization for Economic Cooperation and Development, 2004, 2010). In addition, middle school has emerged as a critical period for determining future success in high school, college, and the workforce (ACT, 2008; Fuller, 2009; Kay, 2009; National Mathematics Advisory Panel, 2008). Finally, the preparation for middle school teachers in the U.S. is the least standardized of any level. Some states have a specialized middle-level certification, others certify middle-level teachers as part of their elementary teacher certification, and still others group middle-level teachers with high school teachers. Thus, the mathematical preparation of middle school teachers varies greatly, with no common standards across the U.S.

To date, nearly all MTCs have been started by teams who have attended a weeklong training workshop on "How to Run a Math Teachers' Circle," organized by AIM and co-sponsored by the NSF, the National Security Agency, and the Mathematical Association of America. The teams generally consist of two mathematicians, two middle-level mathematics teachers, and one additional administrator or organizer. Most of the time, the mathematicians are from the same institute of higher education, which acts as the host institution for administration and grant support. The teachers on the team may be from the same district or different districts, depending on if the MTC is initially aimed at one district or a broader geographic region. Teams spend the week of the workshop participating in sample MTC sessions led by experienced MTC leaders. They also develop concrete plans for their own MTC, including recruitment, meeting logistics, and fundraising for sustainability. After teams leave the workshop, they continue to receive support from the MTC Network through the online resources at www.mathteacherscircle.org, a discussion group, and individual consultation with AIM staff and workshop organizers.

Approximately a year after a team attends the training workshop, they begin their own MTC by holding an intensive summer workshop for approximately 15 to 25 middle-level mathematics teachers. At these intensive summer workshops, the teachers are immersed in doing mathematics guided by several mathematicians and often one or more mathematics educators or teacher-leaders. During the subsequent academic year and following years, the MTC continues to meet approximately once per month either during the week after school hours or on a weekend morning, with summer workshops held as necessary to involve new teachers. The sessions take place at locations chosen by the teams, which vary widely and include both higher education and school settings. Generally, individual sites support MTC activities via some combination of local, foundational, state, and federal funding. Remuneration of participants and facilitators varies depending on the funding sources. For some of the facilitators working in institutions of higher education, their academic year participation is considered part of their duties as a professor, under either the teaching or service categories of their job. Whether or not the participating teachers receive compensation for attending depends on the funding and choices of the individual site, with most sites choosing to provide either professional development credits, course credits, or some form of stipend.

The MTC Model as Professional Development

We chose to use Desimone's (2009) model for professional development as a context for describing the MTC model, as it fit best with our workshop model. Desimone identified five criteria for effective professional development: 1) content focus, 2) active learning, 3) coherence, 4) duration, and 5) collective participation. We argue that the MTC model addresses all five of these criteria. Specifically:

1) Content focus. MTC activities are centered on rich, open-ended problems with multiple entry points. Although the problems can be stated in such a way that a middle or high school student could understand them, some are deep enough that aspects of them are the subject of active mathematical research. Mathematicians are centrally involved in selecting problems and leading sessions to ensure teachers' access to deep content embedded within the mathematical process. Sample problems may be viewed at http://www.mathteacherscircle.org/resources/sessionmaterials.html

2) Active learning. Teachers are involved in active problem solving for the majority of each MTC session, with small group work and whole group discussions occupying the majority of each mathematics session.

3) Coherence. The activities of a MTC are designed to directly support teachers' development of the habits of mind described in the Common Core State Standards - Standards for Mathematical Practice (CCSS-SMP). MTCs intentionally support teachers in developing at least six of the eight CCSS-SMPs, including the ability to (1) make sense of problems and persevere in solving them, (2) reason quantitatively and abstractly, (3) construct viable arguments and critique the reasoning of others, (4) use appropriate tools strategically, (5) attend to precision, and (6) look for and make use of structure. While the specific content addressed in any given MTC varies, the focus on one or more of these critical mathematical practices is always present.

4) Duration. Teachers can participate in MTCs for multiple years, and MTC groups that have met for more than one year report a multi-year retention rate of approximately 70%. Each teacher participates in approximately 30 hours of professional development during the initial intensive summer workshop and between 18 and 24 hours during each academic year of participation.

5) Collective participation. The MTC model builds community among teachers at the same grade levels (6-8). In addition, because the model relies on boundary crossing between professional mathematicians and educators (Ball, 1999), it provides a natural avenue for mathematicians to become involved in K-12 education and form meaningful long-term partnerships with teachers, and for teachers to become part of the larger mathematical community (Beckmann, 2011a, b).

In Desimone's (2009) four-step model for how professional development affects teachers and students, first, teachers participate in professional development that is characterized as effective. Second, this professional development leads to changes in teachers' attitudes, beliefs, knowledge, and/or skills. Third, these changes lead to improved classroom practice, which, finally, leads to increased student learning.

We thus argue that the MTC model can be characterized as effective using Desimone's framework, and we investigate here whether it leads to changes in teachers' mathematical knowledge for teaching, a critical step in the chain toward improved classroom practice and increased student learning.

Mathematical Knowledge for Teaching

A consensus has emerged recognizing that mathematics teachers may need certain types of mathematical knowledge that are specific to the work of teaching mathematics (Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Ball, Thames, & Phelps, 2008). Shulman (1986) defined the concept of pedagogical content knowledge (PCK), which intertwines pedagogy and content, deals with common student difficulties, common student misperceptions, and the utility of different forms of representations in various teaching settings. The concept of PCK was adapted by Ball and colleagues (e.g., Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Ball, Thames, & Phelps, 2008) in a construct known as mathematical knowledge for teaching (MKT). Specifically, mathematical knowledge for teaching refers to the mathematical knowledge that is necessary for the teaching of mathematics (Ball et al., 2008). For example, it is helpful if teachers are able to figure out mathematically how to respond to guide, redirect, or extend students' thinking.

We used items from the Learning Mathematical for Teaching (LMT) instrument (Hill, Schilling & Ball, 2004) as the measure of MKT in our study. Previous studies have reported that higher teacher performance on the LMT is correlated with the presentation of richer mathematics in the classroom (Hill, Ball, & Rowan, 2005; Blunk, 2007; Hill, et al., 2012) and with higher student achievement (Hill, Ball & Rowan, 2005; Rockoff, Jacob, Kane, & Staiger, 2008). Additionally, teachers participating in at least two other professional development programs that emphasize mathematical knowledge for teaching, Developing Mathematical Ideas (http://www.mathleadership.org/page.php?id=33), and Math Recovery (http://www.mathrecovery.org/), have shown significant gains on this instrument (Bell, Wilson, Higgins, & McCoach, 2010; Green & Smith, 2010). In the first, pre-post versions of the LMT instrument were generally offered after two modules of training, with each module consisting of approximately eight three-hour lessons. However, some of the sites used a weeklong workshop model, whereas others were spread out over the semester. Thus, on a calendar scale, the timing of the pre-post offerings differed significantly. For the second study referenced above, the first offering was not until after 60 hours of initial training, and there was a subsequent offering after each of the first two years of training.

Method

We begin with a description of the sites that were used for data collection, and then describe the participants, the instruments used, and the procedure for administering the instruments.

Sites

To be eligible to participate in the study, sites had to plan workshops of at least four full days in length during Summer 2010 and had to target middle-level mathematics teachers as their primary audience. We invited all sites meeting these qualifications to participate, and three of the five eligible sites agreed. Two of these sites (Sites 1 and 3) were just beginning their MTCs in

Summer 2010, and the other site (Site 2) had sustained a MTC continuously since 2006. Although workshop content varied by site, the workshops at all three sites shared several key commonalities: (a) the bulk of the week involved mathematical problem-solving sessions, (b) the participants had ample time to work with each other during these sessions (i.e., lecture was minimal and the majority of time was spent with participants actively engaged in doing mathematics), and (c) most of the mathematical problem-solving sessions were facilitated by active research mathematicians. Additionally, at all three sites, there was no monetary compensation for teachers to attend, nor was there any cost. Two of the sites had a residential workshop format and used grants from local foundations to cover all travel, lodging, and meal costs. The other site had a non-residential workshop that met on the university campus of the host institution, but provided multiple meals and daily snacks, also using a foundation grant. One of the residential sites and the non-residential site also offered tuition for university credit for those teachers who were interested. The workshop facilitators were remunerated for their work during the summer workshop.

Table 1 displays some of the characteristics of each site, including location, length of workshop, whether the workshop was residential, and the number and background of the facilitators. Table 2 gives a breakdown of how much time was spent working on various topics during each workshop. It should be stressed that none of the mathematical sessions directly involved teaching any specific content. Rather, participants were engaged in many aspects of problem solving and problem posing through topics drawn from a variety of content areas. These topics and content areas were chosen based on facilitator preferences and were intentionally balanced to include topics from a variety of different mathematical subdisciplines.

Table 1

Study Site Workshop Characteristics

	Location	Length	Residential?	Facilitators
Site 1	South	5 days	Yes	5 mathematicians, 1 mathematics educator
Site 2	West	4 days	Yes	5 mathematicians, 1 mathematics educator, 1 teacher
Site 3	Southwest	5 days	No	4 mathematicians, 1 teacher

Table 2

Study Site Workshop Topics

	NC	Geo	PS	Other Math	Total Math	Pedagogy	Total
Site 1	4.5*	4.5	3	6	18	4.5	22.5
Site 2	9.5	7	1	6	23.5	4	27.5
Site 3	6	4	0	15	25	3	28

Note. NC = Number concept and operations. Geo = Geometry. PS = Problem-solving strategies. Other Math = Topics primarily focused on other mathematical content areas. Total Math = Total hours spent primarily working on mathematics. Pedagogy = Total hours spent discussing pedagogical topics. Total = Total hours spent on workshop activities (excluding breaks and mealtimes). Many sessions touched on multiple content areas.

*All numbers are hours spent primarily working on the indicated topic.

Participants

A total of 50 teachers across all three sites participated in this study (15 from Site 1, 13 from Site 2, and 22 from Site 3). All were participants in a MTC (a control group was not feasible given the exploratory nature of this study). Of these, forty-five participated in the demographic survey. Survey respondents indicated that their teaching experience ranged from one to 30 years, with an average of 10.5 years of teaching experience (*SD* = 7.3). Twenty-one of the respondents had master's degrees, almost all of which were in education. Thirteen reported having participated in a substantive mathematically oriented professional development program over the past three years, where "substantive" was defined as a program that lasted for a week or more or involved at least 30 hours of the teacher's time. Demographic data broken down by site is provided in Table 3.

Table 3

Participant Demographics

	Site 1	Site 2	Site 3	Combined
Teachers	15	13	22	50
Years Teaching				
Mean (*SD*)	12.0 (7.9)	8.5 (5.5)	11.0 (8.2)	10.5 (7.3)
Range	4 - 28	2 - 22	1 - 30	1 - 30
Master's Degrees	1	8	12	21
Other Substantive PD	3	3	7	13

Note. Master's Degrees = Number of teachers who reported holding a master's degree or higher. Other Substantive PD = Number of teachers who reported having participated in another professional development program within the past 3 years that was primarily focused on mathematics and lasted at least one week or 35 hours.

Instrument

We administered pre- and post-forms of two forms of the LMT instrument—(a) Middle School Number Concepts and Operations, and (b) Middle School Geometry. These two forms were chosen because across the three workshops, they represented the two content areas that received the most attention. Each form of the LMT has undergone extensive testing and revision to establish adequate psychometric soundness (Hill, Schilling & Ball, 2004; Schilling & Hill, 2007), and although the pre and post-test items are different, they are psychometrically equated. The pretest contained 30 items in number concepts and operations, and 19 items in geometry. The posttest contained 33 items in number concepts and operations, and 23 items in geometry.

Due to copyright requirements to maintain the security and hence validity of the instrument, the LMT is not releasable and cannot be included with this paper. However, the authors of the instrument have released several sample items, which can be accessed via their website (http://sitemaker.umich.edu/lmt/home). It should be noted that these items have weaknesses that precluded their inclusion in the final instrument, but they should serve to give the reader a general idea of the genre of questions and type of material included on the LMT.

Procedure

Partway through the morning on the first day of each workshop, participants completed the demographic survey, a survey of mathematical attitudes and beliefs (data not reported here), and the pre-test forms of the Number Concepts and Operations and the Geometry forms of the LMT. Participants were given one hour to complete the two surveys and the LMT instrument. They were instructed to spend no more than one to two minutes on each LMT question. Partway through the final morning of each workshop, participants completed the post-test form of both LMT forms, and also filled out a post-workshop survey asking for their feedback on the workshop activities (survey data not reported here). They were again given one hour to complete the LMT and the survey.

Results

As a condition of using the instrument, we must report standardized scores only. Thus, raw LMT scores are converted to standardized (z) scores using the scoring tables provided at an instrument training workshop that one of the authors attended. These standardized pre-, post- and difference scores are provided in Table 4.

Table 4

Learning Mathematics for Teaching: Standardized Scores

	Number Concepts			Geometry		
	Pre	*Post*	*Difference*	*Pre*	*Post*	*Difference*
Site 1	.35 (1.2)	.59 (1.0)	.24 (.47)	.89 (1.2)	.91 (1.2)	.03 (.47)
Site 2	.67 (.93)	1.1 (.82)	.39 (.55)*	1.3 (.69)	1.5 (.98)	.17 (.89)
Site 3	.08 (.86)	.46 (1.0)	.38 (.55)*	.94 (.86)	1.1 (.89)	.20 (.55)
All	.31 (.98)	.65 (.99)	.34 (.52)*	1.0 (.95)	1.2 (1.0)	.14 (.62)

Note. Scores are standardized and are presented as *M (SD)*. *Pre* = pretest score; *Post* = posttest score; *Difference* = *Post* – *Pre*.
* = Planned comparisons showed a significant difference between pre- and posttest scores ($p < .05$).

The standardized scores were analyzed using a repeated measures ANOVA with two within-subjects factors, Test (Number Concepts and Operations, Geometry) and Time (Pre-test, Post-test), and one between-subjects factor, Site (Workshop Site 1, 2, or 3). We also conducted a series of planned comparison *t*-tests, in order to ascertain whether there were significant differences in pre-test and post-test standard scores at each site and with all sites combined.

Repeated Measures ANOVA

The results of the repeated measures ANOVA are presented in Table 5. The main effect of Site was not significant, $F(2, 46) = 1.22$, $p = .31$, supporting the combination of data across workshop sites. There was a significant main effect of Test, $F(1, 46) = 77.45$, $p < .01$. Overall, Geometry scores were significantly higher than Number Concepts and Operations scores ($M(SD)$ = 1.1(.98) and .48(1.0), respectively), and this pattern was consistent across all three sites. The interaction of Test x Site was not significant, $F(2, 46) = 2.73$, $p = .08$.

There was also a significant main effect of Time, $F(1, 46) = 16.60$, $p < .01$. Posttest scores were higher on average than pretest ones ($M(SD)$ = .90(1.0) and .66(1.0), respectively). The interaction of Time x Site was not significant, $F(2, 46) = .83$, $p = .44$, indicating that all three sites shared a pattern of differences between pre- and post-test administrations and supporting the combination of data across sites for the planned comparison tests.

Overall, there was a larger difference in means for the pre- and post-test scores for Number Concepts and Operations than for Geometry ($M(SE)$ = .34(.52) and .14(.62), respectively). However, the interaction of Test x Time was not statistically significant, $F(1, 46) = 2.59$, $p = .12$. The interaction of Test x Time x Site was also not significant, $F(2, 46) = .01$, $p = .99$. This indicates that the pattern of pre- and post-test scores for each form was not significantly different across sites and further supports combining the data across sites for analysis in the planned comparison tests.

Table 5

Repeated Measures ANOVA: Learning Mathematics for Teaching Standardized Scores

Source	*SS*	*df*	*MS*	*F*	*p*
	Between subjects				
Site	7.99	2	4.00	1.22	.31
Error	151.32	46	3.29		
	Within subjects				
Test	15.65	1	15.65	77.46**	.00
Test x Site	1.11	2	0.55	2.73	.08
Test (Error)	9.30	46	0.20		
Time	2.53	1	2.53	16.60**	.00
Time x Site	0.25	2	0.13	0.83	.44
Time (Error)	7.01	46	0.15		
Test x Time	0.48	1	0.48	2.59	.12
Test x Time x Site	0.01	2	0.00	0.01	.99
Test x Time (Error)	8.48	46	0.18		

Note. Site = Workshop Site 1, 2, or 3. Test = Number Concepts and Operations or Geometry. Time = Pretest or Posttest.
$^{**}p < .01$.

Planned Comparisons

The planned comparison *t*-tests revealed that the increase in the Number Concepts and Operations scores was significant with all sites combined, $M(SD) = .34(.52)$, $t(48) = 4.56$, $p < .001$. Sites 2 and 3 each also demonstrated a significant increase on Number and Operations ($M(SD) = .39(.56)$, $t(11) = 2.45$, $p = .03$; and $M(SD) = .38(.52)$, $t(21) = 3.27$, $p = .004$; respectively). Site 2's scores also increased ($M(SD) = .24(.47)$), but the difference in pre- and post-test scores was not significant, $t(14) = 1.92$, $p = .08$.

The change in LMT scores was not significant for the Geometry form with all sites combined, $M(SD) = .14(.62)$, $t(48) = 1.59$, $p = .12$. None of the sites showed significant changes in Geometry scores ($M(SD) = .03(.47)$, $t(14) = .23$, $p = .82$; $M(SD) = .17(.89)$, $t(11) = .65$, $p = .53$; and $M(SD) = .20(.55)$, .$t(21) = 1.72$, $p = .10$; for Sites 1, 2, and 3, respectively).

Discussion and Conclusions

This study represents the first attempt to characterize some of the effects of MTC participation on teachers. In particular, we examined the research question of whether participating in a four- to five-day intensive MTC workshop affected teachers' mathematical knowledge for teaching as measured by a standard instrument for assessing this, the LMT. The results indicate that across three sites, teachers' scores on the Number Concepts and Operations form of the LMT instrument increased significantly over the course of the workshop. The mean score increase for Number Concepts and Operations was .34, or around a third of a standard unit. Each of the sites did spend a large proportion of time on problems related to number concepts and operations; this ranged from 24% (6 out of 25 hours) at Site 3 to 40% (9.5 out of 23.5 hours) at Site 2. However, in all cases, teachers were spending fewer than 10 hours on related topics, and there was no attempt to match specific test items with topics covered. Given how little time the teachers spent on topics related to the Number Concepts and Operations form, our preliminary hypothesis, to be explored in future work, is not that teachers were gaining additional content knowledge in these areas, but rather that the experience of participating in the MTC workshop enabled them to reason more deeply about content knowledge that they already possessed. In particular, we hypothesize that the intensive time spent and hence experience gained at engaging in the practice of doing mathematics is the catalyst for this increased mathematical reasoning skill.

The pre-test/post-test design employed in this study, although valuable as a first step toward examining the effects of MTC participation on teachers' mathematical knowledge for teaching, provides limited insight into the mechanism behind these effects. A combination of additional quantitative data and qualitative data will be necessary to gain further understanding of how MTCs affect teachers' mathematical knowledge for teaching and whether these effects translate into their classroom teaching. With a larger number of study participants, we can examine interactions between teacher and site demographics and LMT scores. Qualitative data from teacher surveys, interviews, and classroom observations will provide additional insights and supporting evidence regarding the effects of MTC participation.

For comparison, a recent study (Bell et al., 2010) examined the effects of the Developing Mathematical Ideas (DMI) professional development program on Mathematical Knowledge for Teaching at 10 sites nationally. DMI consists of seven modules of well-developed curriculum that focuses on the mathematics of K-8 classrooms. Each module was designed to be presented in eight 3-hour sessions, for a total of 24 hours per module. The sessions consist of time for teachers to work problems from a specific mathematical strand and to explore classroom connections through videos of classroom instruction and examining student work. Bell and her colleagues studied 10 sites that implemented two modules focusing on the number and operations strand (Building a System of Tens and Making Meaning for Operations), for a total of 48 hours of professional development focused on number and operations at each site, completed over an average of 5 months (completion time varied considerably by site, from 10 days to 1 year). Bell and her colleagues administered items from the Number Concepts and Operations form of the LMT to teachers at each site before and after they had completed the DMI modules, choosing the LMT items to match as closely as possible with the content covered. Across the 10 sites, they reported an average score increase of .35 standard units for teachers participating in DMI. Here we are reporting an average gain of .34 standard units on a version of the Number Concepts and Operations form that was not customized, for teachers who participated in a professional development program that was not tailored to classroom content and in which teachers spent much less time on content related to number and operations.

Although there was a slight average increase in Geometry scores (.14 standard units), this increase did not represent a significant gain on the Geometry form for any one site or overall. Because Geometry and Number Concepts and Operations represented a comparable amount of the content at all three sites, it does not appear likely that time spent on the content was the critical difference. We hypothesize that instead, a difference is not apparent because of a ceiling effect in these teachers' Geometry scores. Supporting this argument, Geometry scores on the pretest averaged 1.0 standard units, indicating that teachers were already well above average performance on this form, and scores were also significantly higher overall than Number Concepts and Operations scores, by an average of .58 standard units. We are unable to hypothesize about the reasons for this at this time. Given this possible ceiling effect, it is likely that we will explore using other forms of the LMT instruments instead of Geometry in future work.

This study provides preliminary evidence that MTCs may be effective in increasing teachers' mathematical knowledge for teaching, at least in some content areas. The improvement in teachers' LMT scores for Number Concepts and Operations suggests that MTCs at least partially meet the second step of Desimone's (2009) four-step model for how professional development affects teachers and students, that the professional development leads to changes in teachers' attitudes, beliefs, knowledge, and/or skills. Because MTCs are not focused on addressing particular content related to the school curriculum but rather focus on developing teachers as mathematicians, we hypothesize that the mechanism behind this increase in LMT scores may be improved facility with the practice of mathematics, which leads to increased ability to reason about known content. Further inquiry is needed to investigate this hypothesis. In future work, we will also begin to turn our attention to the third step of Desimone's model by investigating whether participation in MTCs is associated with changes in teachers' classroom practice, and what specific aspects of teaching are affected.

The fact that LMT scores improved across sites is encouraging given that at two of the three sites, the workshops were conducted by facilitators who were not originators of the MTC model and who had learned about the model through training workshops. This study thus provides preliminary evidence that the MTC model can be effectively scaled up to multiple sites with multiple facilitators. To our knowledge, this study and Bell et al. (2010) are the only ones to date to investigate how a single professional development program for classroom teachers implemented at multiple sites with multiple facilitators affects mathematical knowledge for teaching. According to Borko (2004), this type of study is critical for learning more about the interactions among professional development programs, facilitators, and teachers. It is hoped that future work on MTCs will continue to contribute to our knowledge of how professional development programs can be effectively brought to scale.

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